THE DEFINITE INTEGRAL

OBJECTIVE: MAKE OUR DISCUSSION FROM WEDNESDAY MATHEMATICALLY PRECISE

Def WHEN $f(x)$ IS POSITIVE AND $a<b$ THEN...

THE AREA UNDER THE GRAPH OF f BETWEEN $x=a$ AND $x=b$ $:=$ $\int_a^b f(x)\,dx$

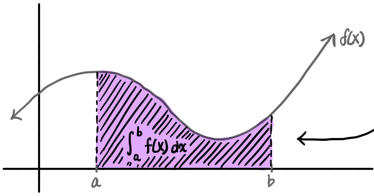
ELONGATED $\int$ INDICATES INTEGRATION

a & b ARE CALLED THE LIMITS OF INTEGRATION

dx: THIS TELLS US WHAT WE'RE INTEGRATING WITH RESPECT TO

$f(x)$: FUNCTION YOU'RE INTEGRATING IS CALLED THE INTEGRAND

THIS IS CALLED A **DEFINITE INTEGRAL**



AN ILLUSTRATION OF AN INTEGRAL.

Def LET $f(x)$ BE A FUNCTION, AND LET $g(x)$ BE SUCH THAT $g'(x)=f(x)$.

THEN, WE SAY THAT $g(x)$ IS AN **ANTIDERIVATIVE** OF $f(x)$.

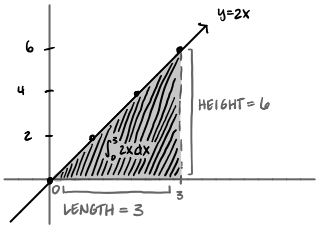
IN THIS PARTICULAR COURSE.. ANTIDIFFERENTIATION = INTEGRATION = "UNDOING" DIFFERENTIATION

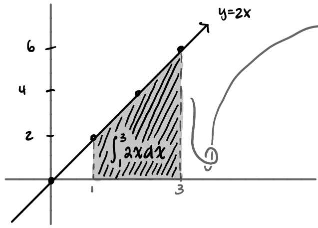
Ex. AN ANTIDERIVATIVE OF $2x$ is x^2.

THIS IS BECAUSE $\frac{d}{dx}(x^2)=2x$!

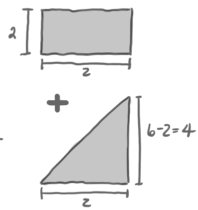
But what does this have to do with the area under a curve?

let's investigate...





AREA...



$\frac{1}{2}(4)(2)=4$

$\text{AREA} = \int_0^3 2x\,dx = \frac{1}{2}(3)(6) = \frac{1}{2}(18) = \underline{\underline{9}}$

$\text{AREA} = \int_1^3 2x\,dx = 4+4 = \underline{\underline{8}}$

NOTICE THE FOLLOWING:

IF WE TAKE x^2 EVALUATED AT $x=0$ AND $x=3$...

$(3)^2-(0)^2 = \underline{9}.$

so somehow, these are related!

IF WE TAKE x^2 EVALUATED AT $x=1$ AND $x=3$...

$(3)^2-(1)^2 = 9-1 = \underline{\underline{8}}$

Def. RECALL THE PROBLEM FROM WEDNESDAY. WE ESTIMATED THE DISTANCE THAT GOGO RAN BY FINDING THE AREA OF RECTANGLES UNDER THE CURVE, WHICH WE CALLED UPPER & LOWER SUMS. (ALTERNATIVELY, RIGHT-HAND SUMS & LEFT-HAND SUMS RESPECTIVELY.) WE NOW MAKE THIS IDEA MATHEMATICALLY PRECISE.

TO ESTIMATE $\int_a^b f(x)\,dx$, WE DIVIDE THE INTERVAL $[a,b]$ INTO n EQUAL SUBDIVISIONS.

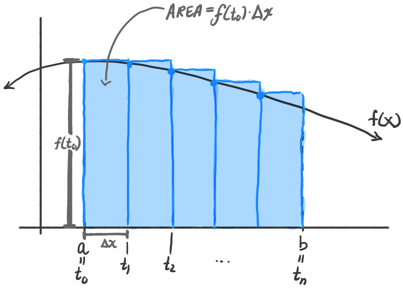
WE SAY THE WIDTH OF EACH SUBDIVISION IS $\Delta x = \dfrac{b-a}{n}$

LET $t_0, t_1, \ldots, t_n$ THE ENDPOINTS OF THE SUBDIVISIONS.

THEN, THE LEFT & RIGHT-HAND SUMS WILL BE:

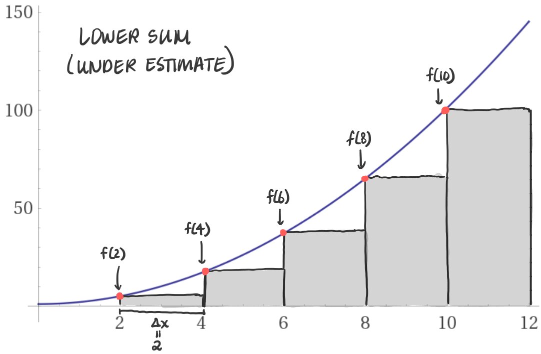
$$\text{LEFT SUM} = f(t_0)\Delta x + f(t_1)\Delta x + \ldots + f(t_{n-1})\Delta x$$

$$\text{RIGHT SUM} = f(t_1)\Delta x + f(t_2)\Delta x + \ldots + f(t_n)\Delta x$$



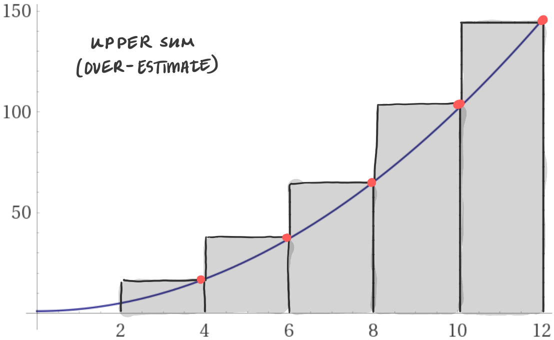
(AN EXAMPLE OF A LEFT-HAND SUM)

EXAMPLE USING UPPER & LOWER SUMS, ESTIMATE $\int_2^{12} x^2+1\,dx$



FOR THE LOWER SUM, OUR ESTIMATE WILL BE:

$f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2$



THE UPPER SUM WILL BE:

$f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2 + f(12)\cdot 2$

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