

# DISTANCE & ACCUMULATED CHANGE

(SECTION 5.1)

OBJECTIVE(S):

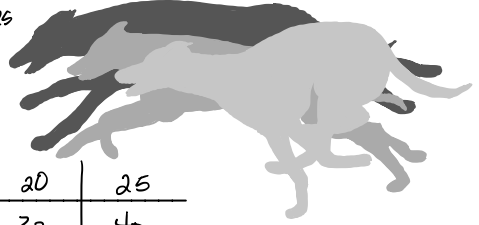
INVESTIGATE HOW VELOCITY & DISTANCE ARE RELATED; IN PARTICULAR HOW TO FIND DISTANCE FROM VELOCITY

IN THE PREVIOUS CHAPTER(S) WE USED THE DERIVATIVE TO FIND THE RATE OF CHANGE OF A GIVEN FUNCTION.

WHAT IF WE'RE GIVEN THE RATE OF CHANGE, BUT WANT TO RECOVER THE ORIGINAL FUNCTION?

$$\text{DISTANCE} = \text{VELOCITY} \times \text{TIME}$$

Ex SUPPOSE WE RECORD GOGO RACING. IT APPEARS AS THOUGH HER VELOCITY IS INCREASING; WE MEASURE HER VELOCITY (SPEED+DIRECTION) AT 5 SECOND INTERVALS:



TIME (SECONDS)	0	5	10	15	20	25
VELOCITY (FT/SEC)	5	10	17	27	32	40

WE DON'T KNOW HOW FAST GOGO IS MOVING EVERY SECOND, SO WE CAN'T FIND THE EXACT DISTANCE THAT SHE RAN... BUT WE CAN ESTIMATE IT!

## GETTING A LOWER ESTIMATE

- GOGO IS GOING AT LEAST 5 ft/SECOND FOR THE FIRST 5 SECONDS.

SINCE DISTANCE = VELOCITY  $\times$  TIME, WE SEE SHE TRAVELED AT LEAST 25 ft IN THE FIRST FIVE SECONDS.

- SO... DURING THE 25-SECOND PERIOD ON THE CHART ABOVE, GOGO TRAVELS AT LEAST.

$$(5 \cdot 5) + (10 \cdot 5) + (17 \cdot 5) + (27 \cdot 5) + (32 \cdot 5) = \underline{455 \text{ ft}}$$

## GETTING A HIGHER ESTIMATE

- WE CAN ALSO OVERESTIMATE THE DISTANCE THAT GOGO TRAVELED.

DURING THE FIRST 5 SECONDS, GOGO IS GOING AT MOST 10 ft/SEC.

THUS, SHE TRAVELED AT MOST  $5 \cdot 10 = 50 \text{ ft}$  IN THE FIRST 5 SECONDS.

- DURING THE 25-SECOND PERIOD OF THE CHART, GOGO TRAVELS AT MOST

$$(5 \cdot 10) + (5 \cdot 17) + (5 \cdot 27) + (5 \cdot 32) + (5 \cdot 40) = \underline{630 \text{ ft}}$$

so far..

$$455 \text{ ft} \leq \text{total distance Gogo travelled} \leq 630 \text{ ft}$$

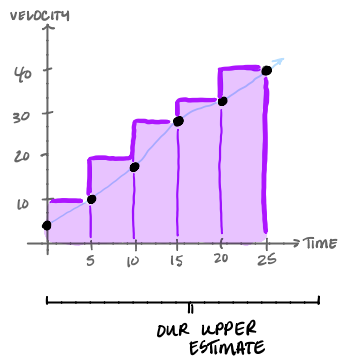
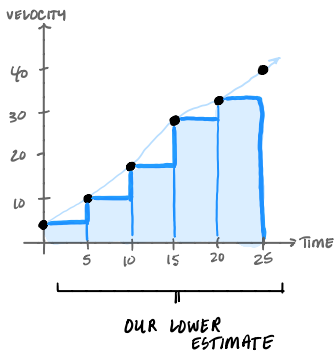
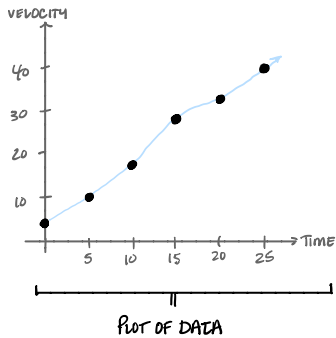
THIS IS AN UNDERESTIMATE OF THE TOTAL DISTANCE GOGO TRAVELED.

THIS IS AN OVERESTIMATE OF THE TOTAL DISTANCE GOGO TRAVELED.

TO GET AN EVEN BETTER ESTIMATE, MAYBE WE COULD AVERAGE OUR ESTIMATES TOGETHER.

$$\frac{455 + 630}{2} = 542.5 \text{ ft}$$

GRAPHICALLY, WHAT WE'VE DONE IS THE FOLLOWING:



FACT: IF VELOCITY IS POSITIVE, THE TOTAL DISTANCE TRAVELLED IS THE AREA UNDER THE VELOCITY CURVE.

EXAMPLE. ALICIA IS RIDING A BICYCLE. HER VELOCITY, IN FEET/SEC IS GIVEN BY  $v(t) = 5t$ .  
HOW FAR DOES BIKE TRAVEL IN 3 SEC?