LOCAL MAXIMA & MINIMA

SECTION 4.1 IN THE TEXT

OBJECTIVES:

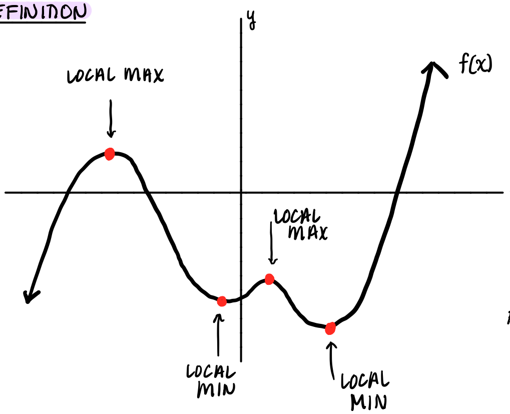
- DEFINE LOCAL MINS/MAXS, & DEVISE A METHOD TO FIND THEM

RECALL THAT THE DERIVATIVE $f'(x)$ OF A FUNCTION $f(x)$ TELLS US THE FOLLOWING:

- WHENEVER $f'(x)>0 \rightarrow f$ INCREASING
- WHENEVER $f'(x)<0 \rightarrow f$ DECREASING
- WHENEVER $f'(x)=0 \rightarrow f$ CONSTANT

OFTEN TIMES, WE WANT TO FIGURE OUT WHERE A FUNCTION IS SMALLEST OR LARGEST RELATIVE TO NEARBY POINTS. (SUCH AS MAXIMIZING PROFIT, OR MINIMIZING COST)

DEFINITION



- WE SAY THAT f HAS A **LOCAL MIN** AT THE POINT $x=p$ IF $f(p) \leq$ VALUES OF f FOR POINTS AROUND $x=p$.
- WE SAY THAT f HAS A **LOCAL MAX** AT THE POINT $x=p$ IF $f(p) \geq$ VALUES OF f FOR POINTS AROUND $x=p$.

? SOO.. HOW DO WE DETECT LOCAL MAXS & MINS? ?

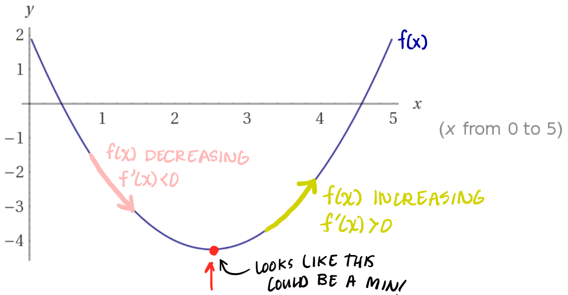
DEFINITION ANY POINT $x=p$ WHERE $f'(p)=0$ OR $f'(p)$ IS UNDEFINED IS CALLED A **CRITICAL POINT** OF f.

FACT: IF A FUNCTION HAS A LOCAL MIN OR MAX AT $x=p$, THEN $x=p$ IS A CRITICAL POINT!

CAUTION! IF $x=p$ IS A CRITICAL POINT, IT IS NOT NECESSARILY A LOCAL MAX OR LOCAL MIN

Ex. FIND LOCAL MINS & MAXS OF $f(x) = x^2 - 5x + 2$

GOAL: FIND ALL CRITICAL POINTS OF $f(x)$
THEN TRY TO DETERMINE IF THEY ARE MAXIMA/MINIMA.



BY DEFN, CRITICAL POINTS ARE POINTS $x=p$ WHERE $f'(p)=0$. SO, LET'S FIND $f'(x)$ AND SET IT EQUAL TO ZERO.

$$f'(x) = 2x - 5$$

$f'(x)=0$ HERE

WE'LL SET $f'(x)=0$ AND SOLVE FOR x.

$f'(x) = 2x - 5 = 0 \longrightarrow 2x = 5 \longrightarrow \boxed{x = 5/2.}$ A CRITICAL POINT!

LOOK AT $f'(x)$ AROUND $x = 5/2$... HOW DOES IT BEHAVE?

HOW CAN WE DETERMINE IF $x = 5/2$ IS A LOCAL MIN OR MAX?

* NOTICE: TO THE LEFT OF $x = 5/2$, $f(x)$ DECREASING $\Rightarrow f'(x)<0$
TO THE RIGHT OF $x = 5/2$, $f(x)$ INCREASING $\Rightarrow f'(x)>0$

$f(x)$ GOES FROM DECREASING TO INCREASING AT $x = 5/2$
$\Rightarrow$ $\underline{x = 5/2 \text{ IS A LOCAL MIN OF } f(x)}$

ALTERNATIVELY, WE CAN SAY THAT $f'(x)$ GOES FROM NEGATIVE TO POSITIVE AROUND $x=5/2$

THE FIRST DERIVATIVE TEST FOR LOCAL MAXIMA & MINIMA

LET $x=p$ A CRITICAL POINT OF $f(x)$. ($f'(p)=0$ OR $f'(p)$ UNDEFINED)

- IF $f'(x)$ CHANGES FROM $-$ TO $+$ AROUND $x=p$, THEN $f(x)$ HAS A **LOCAL MINIMUM** AT $x=p$.
- IF $f'(x)$ CHANGES FROM $+$ TO $-$ AROUND $x=p$, THEN $f(x)$ HAS A **LOCAL MAXIMUM** AT $x=p$.

Ex. FIND ANY MAXIMA OR MINIMA OF $f(x) = x^5(2-x)^6$.
(WE'LL WANT TO USE THE FIRST DERIVATIVE TEST)

STEP 1 FIND $f'(x)$.
FOR THIS WE NEED PRODUCT & CHAIN RULE.

$$f(x) = \underset{h(x)}{\underset{\downarrow}{x^5}}\underset{g(x)}{\underset{\downarrow}{(2-x)^6}}$$

PRODUCT RULE:

$h(x) = x^5$ $\quad$ $h'(x) = 5x^4$

$g(x) = (2-x)^6$ $\quad$ $g'(x)$ = CHAIN RULE $= -6(2-x)^5$

$$f'(x) = -6x^5(2-x)^5 + 5x^4(2-x)^6$$

STEP 2 SET $f'(x)=0$, SOLVE FOR x

WE HAVE: $\quad -6x^5(2-x)^5 + 5x^4(2-x)^6 = 0$

$$x^4(2-x)^5\left(-6x + 5(2-x)\right) = 0$$

$$x^4(2-x)^5\left(-6x + 10 - 5x\right) = 0$$

$$x^4(2-x)^5(-11x+10) = 0$$

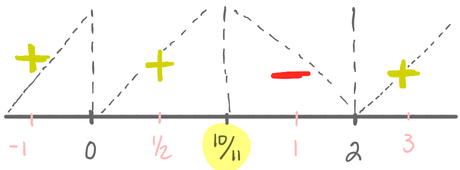
NOW AT THIS POINT WE CAN SET EACH FACTOR $= 0$ TO GET SOLUTIONS FOR x

- $x^4 = 0 \longrightarrow x = 0$
- $2-x = 0 \longrightarrow x = 2$
- $-11x+10 = 0 \longrightarrow x = 10/11$

These are my critical points!

STEP 3 DETERMINE BEHAVIOR OF $f'(x)$ AROUND CRITICAL POINTS.

I LIKE TO USE A NUMBER LINE TO DO THIS.



CHOOSE TEST POINTS AROUND EACH OF THE CRITICAL POINTS

USE THESE TO FIGURE OUT IF $f'(x)$ IS POS. OR NEGATIVE!

$$f'(-1) = \underset{+}{(-1)^4}\,\underset{+}{(2-(-1))^5}\,\underset{+}{(-11(-1)+10)} \rightsquigarrow +$$ $f'(-1)$ IS POSITIVE

$$f'(1/2) = \underset{+}{(1/2)^4}\,\underset{+}{(2-(1/2))^5}\,\underset{+}{(-11(1/2)+10)} \rightsquigarrow +$$ $f'(1/2)$ IS POSITIVE

$$f'(1) = \underset{+}{(1)^4}\,\underset{+}{(2-(1))^5}\,\underset{-}{(-11(1)+10)} \rightsquigarrow -$$ $f'(1)$ IS NEGATIVE

$$f'(3) = \underset{+}{(3)^4}\,\underset{-}{(2-(3))^5}\,\underset{-}{(-11(3)+10)} \rightsquigarrow +$$ $f'(3)$ IS POSITIVE

⇒ $x = 10/11$ IS A LOCAL MAXIMUM OF $f(x)$!

⇒ $x=2$ IS A LOCAL MINIMUM OF $f(x)$!