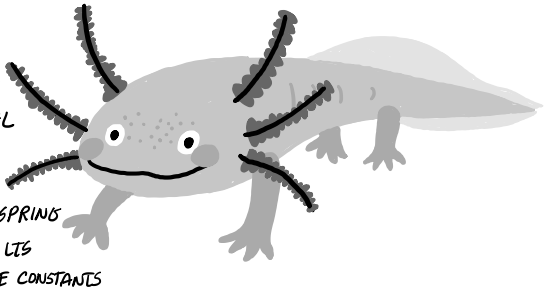
APPLICATIONS OF DERIVATIVES

EXAMPLE (THE RICKER CURVE)

THE NUMBER OF OFFSPRING IN A POPULATION IS NOT NECESSARILY A LINEAR FUNCTION OF THE NUMBER OF ADULTS. THE RICKER CURVE, USED TO MODEL FISH POPULATIONS IN THE SETTING OF A LABORATORY, CLAIMS THAT:

$$y = axe^{-bx}$$ WHERE y = # OF OFFSPRING, x = # OF ADULTS, a, b ARE POSITIVE CONSTANTS



(A) FIND & CLASSIFY THE CRITICAL POINT OF THE RICKER CURVE

RECALL: $x=a$ IS A CRITICAL POINT OF $f(x)$ WHEN $f'(a)=0$ (OR IS UNDEFINED)

WE'LL USE THE FIRST DERIVATIVE TEST.

STEP 1 FIND y'. TO DO SO IN THIS CASE WE'LL USE THE PRODUCT RULE & CHAIN RULE.

axe^{-bx}

$f(x) = ax$ $\quad f'(x) = a$

$g(x) = e^{-bx}$ $\quad g'(x) =$ CHAIN RULE $= -be^{-bx}$

CHAIN RULE: e^{-bx}

$f(x) =$ OUTSIDE $= e^x$ $\quad f'(x) = e^x$

$g(x) =$ INSIDE $= -bx$ $\quad g'(x) = -b$

$\Big\} = -be^{-bx}$

$$y' = ae^{-bx} - abxe^{-bx}$$

STEP 2 SET $y'=0$ AND SOLVE FOR x.

It might help to try to factor this

$$ae^{-bx} - abxe^{-bx} = 0$$

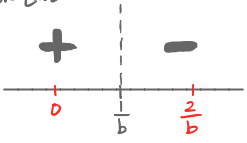
↓ factor

$$ae^{-bx}(1 - bx) = 0$$

$$1 - bx = 0 \Rightarrow -bx = -1 \text{ so } \boxed{x = \frac{1}{b}}$$

SO OUR CRITICAL POINT IS $x = \frac{1}{b}$.

STEP 3 NUMBER LINE



WE CAN CHOOSE OUR TEST POINTS TO BE $x=0$ AND $x=\frac{2}{b}$.

- $y'(0) = ae^{-b(0)} - ab(0)e^{-b(0)} = a > 0 \longrightarrow +$
- $y'(2/b) = ae^{-b(2/b)} - ab(2/b)e^{-b(2/b)} = ae^{-2} - 2ae^{-2} < 0 \longrightarrow -$

THUS, BY THE FIRST DERIVATIVE TEST, $x=\frac{1}{b}$ IS A MAXIMUM OF THE RICKER CURVE.

(B) FIND GLOBAL MAXIMUM

THIS SHOULD OCCUR AT $x=1/b$. (WE'RE JUST LOOKING FOR THE VALUE OF THE FUNCTION AT $x=1/b$)

SO.. WE HAVE

$$y = axe^{-bx} \quad \text{so} \quad y(1/b) = a(1/b)e^{-b(1/b)}$$

$$= \frac{a}{b}e^{-1}$$

SO THE GLOBAL MAXIMUM OCCURS AT

$$\left(\frac{1}{b}, \frac{a}{b}e^{-1}\right)$$