DERIVATIVES

RECALL THAT WE'VE DEFINED THE DERIVATIVE OF f AT $x=a$, $f'(a)$, AS THE INSTANTANEOUS RATE OF CHANGE OF THE FUNCTION $f(x)$ AT $x=a$.

DEFN. LET $f(x)$ SOME (DIFFERENTIABLE) FUNCTION DEFINED AT THE POINT $x=a$. THE **TANGENT LINE** OF $f(x)$ AT POINT a IS A LINE $y=mx+b$ WITH $m=f'(a)$ (IT'S A LINE WITH SLOPE $f'(a)$) THAT PASSES THROUGH THE POINT $(a, f(a))$.

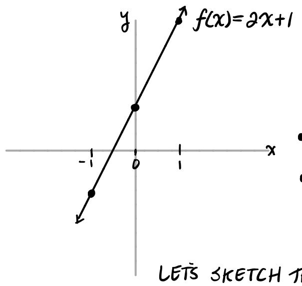
CONCEPTUALLY, THIS IS THE LINE WE'LL GET IN THE TEXTBOOK/WINDOW EXAMPLE WHEN WE LET Δt BECOME ARBITRARILY CLOSE TO ZERO.

ALSO RECALL THAT IF WE HAVE A LINE $y=mx+b$, THE SLOPE m TELLS US INFORMATION ABOUT WHETHER THE LINE IS INCREASING, DECREASING, OR CONSTANT.

IN GENERAL, FOR A DIFFERENTIABLE FUNCTION $f(x)$...

- WHENEVER $f'(x)>0$ ON (a,b), $f(x)$ IS **INCREASING**
- WHENEVER $f'(x)<0$ ON (a,b), $f(x)$ IS **DECREASING**
- WHENEVER $f'(x)=0$ ON (a,b), $f(x)$ IS **CONSTANT**

EX. SKETCH THE GRAPH OF $f'(x)$ GIVEN THAT $f(x)=2x+1$.



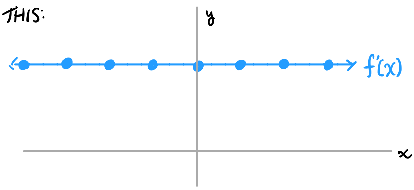
TO SKETCH $f'(x)$ IT MIGHT BE HELPFUL TO ASK OURSELVES:

- WHAT IS THE SLOPE OF $f(x)$ AT $x=-1$?
- WHAT IS THE SLOPE OF $f(x)$ AT $x=0$?
- WHAT IS THE SLOPE OF $f(x)$ AT $x=1$?

THESE ARE SORT OF SILLY QUESTIONS, $f(x)$ IS A LINE WITH SLOPE $m=2$...

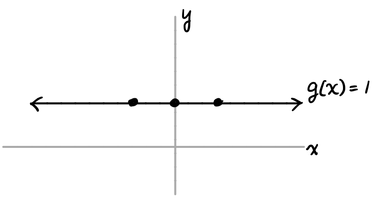
SO $f'(x)=2$ FOR EVERY VALUE OF x!

LET'S SKETCH THIS:



$\boxed{\text{SO } f'(x)=2.}$

EX. SKETCH THE GRAPH OF $g'(x)$ GIVEN THAT $g(x)=1$.



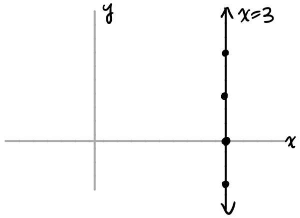
- SO.. WHAT'S THE SLOPE OF $g(x)$ AT A GIVEN VALUE OF x?

$(x_1, 1)$ & $(x_2, 1)$ ARE POINTS ON $g(x)$.

$$\frac{\Delta y}{\Delta x} = \frac{1-1}{x_2-x_1} = 0.$$

$\boxed{\text{SO } g'(x)=0}$

Ex. WHAT ABOUT VERTICAL LINES LIKE $x = 3$?



WELL.. $(3, y_1)$ & $(3, y_2)$ ARE POINTS ON $x=3$.. SO

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{3-3} = \frac{y_2 - y_1}{0}$$

CAN WE COME UP WITH A GENERAL FORMULA FOR $f'(x)$ IN THE PREVIOUS EXAMPLES?

- DERIVATIVE RULE FOR $y = f(x) = mx+b$ ...

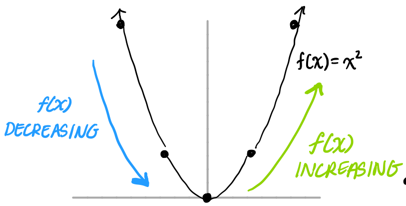
$$f'(x) = m \quad \text{OR WE CAN WRITE} \quad \frac{dy}{dx} f(x) = m$$

- DERIVATIVE RULE FOR $y = f(x) = c$, WHERE c IS A CONSTANT, WE HAVE:

$$f'(x) = 0 \quad \text{OR WE CAN WRITE} \quad \frac{dy}{dx} f(x) = 0$$

Ex WHAT ABOUT OTHER POWERS OF x?

SAY $f(x) = x^2$...



HOW COULD I SKETCH $f'(x)$?

NOTICE: TO THE LEFT OF $x=0$.. $f(x)$ IS DECREASING, TO THE RIGHT OF $x=0$, $f(x)$ IS INCREASING.

SO:

- ON $(-\infty, 0)$ $f'(x) < 0$

(THIS IS WHAT DECREASING MEANS!)

- ON $(0, \infty)$ $f'(x) > 0$

(THIS IS WHAT INCREASING MEANS)

... BUT WHAT'S GOING ON AT $x=0$?

A CLOSER LOOK...