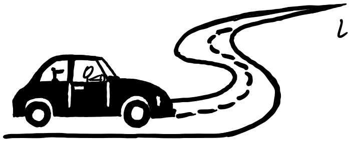
INSTANTANEOUS RATE OF CHANGE



LET'S SAY YOU'RE DRIVING, AND YOUR POSITION IS GIVEN BY

$$y = f(t) \qquad \left(\begin{array}{l}\text{WITH } t = \text{time (hrs)} \\ y = \text{distance (mi)}\end{array}\right)$$

WE KNOW FROM BEFORE THAT YOUR AVERAGE RATE OF CHANGE IN POSITION FROM TIME $t=a$ TO $t=b$ IS GIVEN BY:

$$A.R.C = \frac{\Delta y}{\Delta t} = \frac{f(b)-f(a)}{b-a}$$

ALSO RECALL: A.R.C OF DISTANCE W.R.T TIME IS AVERAGE VELOCITY

SO... IF YOU DRIVE 200mi IN 4 HOURS, YOUR AVERAGE VELOCITY WAS 50MPH.

EXACTLY AS YOU'VE DRIVEN 200 MILES, YOU GET PULLED OVER FOR SPEEDING IN A 60MPH ZONE.
THERE'S NO WAY YOU COULD HAVE BEEN SPEEDING IF OUR AVERAGE VELOCITY WAS 50MPH

... RIGHT?

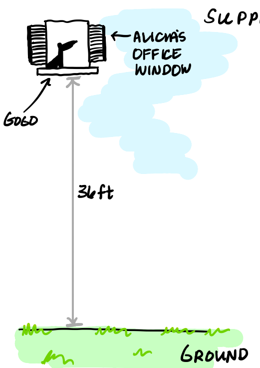
NO. MY AVERAGE VELOCITY BETWEEN TIME a AND b DOESN'T TELL ME ANYTHING ABOUT WHAT HAPPENED BETWEEN $t=a$ AND $t=b$.

TO SAY "MY AVG VELOCITY WAS 50MPH" DOESN'T MEAN YOU WENT 50MPH THE ENTIRE TRIP.

YOUR VELOCITY AT A GIVEN TIME t DURING YOUR TRIP IS WHAT'S SHOWN ON THE SPEEDOMETER OF YOUR CAR.

THIS IS WHAT I'M INTERESTED IN.

Ex ALICIA THROWS A CALCULUS TEXTBOOK OUT OF HER OFFICE WINDOW, WHICH IS 36ft FROM THE GROUND. WE MEASURE THE HEIGHT y (IN FEET) OF THE TEXTBOOK ABOVE THE GROUND AT TIME t (SECONDS)



SUPPOSE THAT WE OBSERVE THAT THE EQUATION

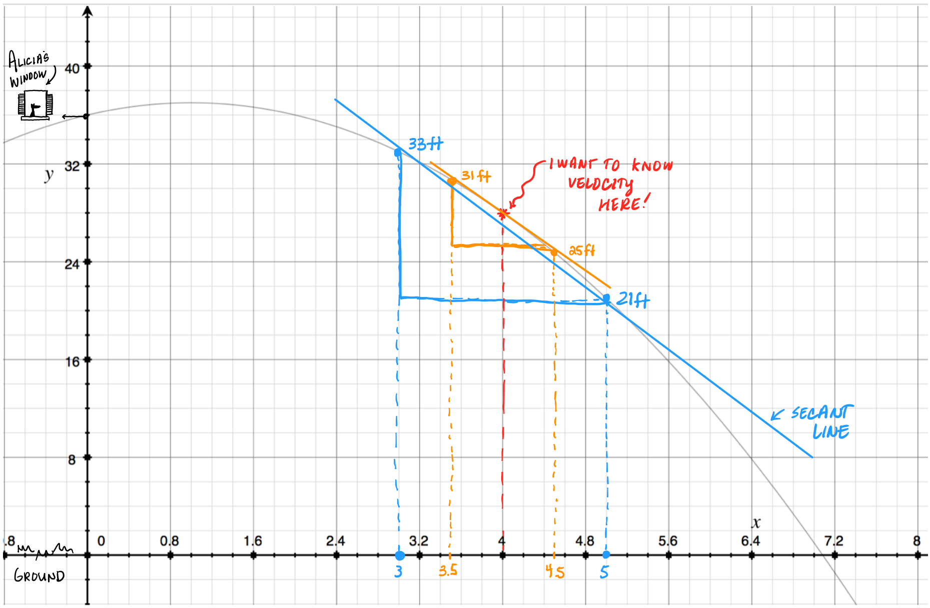
$$y = -t^2 + 2t + 36$$

PERFECTLY MODELS THIS SITUATION.

ALICIA CLAIMS THAT AT $t=4$, THE VELOCITY OF THE TEXTBOOK IS -4 ft/sec. (WHY IS IT NEGATIVE?)

GOAL: DETERMINE IF ALICIA'S CLAIM IS TRUE.

TO DO SO.. WE CAN START BY FINDING THE AVG RATE OF CHANGE OF INTERVALS AROUND $t=4$.



FROM $t=3$ TO $t=5$

$(\Delta t = 5-3 = 2)$

$(3, 33)$ $\quad (5, 21)$

$t_1 \quad h_1 \qquad t_2 \quad h_2$

AVG RATE OF CHANGE FROM $t=3$ TO $t=5$..

$$= \frac{h_2 - h_1}{t_2 - t_1} = \frac{21-33}{5-3} =$$

LET'S MOVE IN CLOSER TO $t=4$...

$(\Delta t = 4.5 - 3.5 = 1)$

FROM $t=3.5$ TO $t=4.5$

AVG RATE OF CHANGE FROM $t=3.5$ TO $t=4.5$

$(3.5, 31)$ $\quad (4.5, 25)$

$t_1 \quad h_1 \qquad t_2 \quad h_2$

$$= \frac{h_2 - h_1}{t_2 - t_1} = \frac{25-31}{4.5-3.5} =$$

EVEN CLOSER...

$(\Delta t = 4 - 3.999 = .001)$

FROM $t=3.999$ TO $t=4.001$

AVG RATE OF CHANGE FROM $t=3.999$ TO $t=4.001$

$(3.999, f(3.999))$ $\quad (4.001, f(4.001))$

$t_1 \quad h_1 \qquad t_2 \quad h_2$

$$= \frac{h_2 - h_1}{t_2 - t_1}$$

NOTICE THAT WE'RE LOOKING AT WHAT HAPPENS AS Δt GETS CLOSER TO ZERO!

THE CLOSER OUR INTERVAL BECOMES TO $t=4$, THE CLOSER OUR ESTIMATE WILL BE TO THE ACTUAL VELOCITY @ $t=4$!

LET f BE SOME FUNCTION → FORMALLY WE NEED RESTRICTIONS ON WHAT f CAN BE... NOT ALL FUNCTIONS ARE DIFFERENTIABLE!

<u>DEFN</u>. THE <u>DERIVATIVE OF f AT a</u>, WRITTEN $f'(a)$, IS THE INSTANTANEOUS RATE OF CHANGE OF f AT POINT a.

- IT'S THE "SLOPE" OF f AT THE POINT a
- IT'S THE SLOPE OF THE <u>TANGENT</u> LINE TO THE CURVE f AT THE POINT a.

WE CAN ALSO WRITE THIS USING <u>LEIBNIZ NOTATION</u>: $f'(x) = dy/dx$.

WE APPROXIMATE $f'(x)$ USING $\Delta y/\Delta x$.. THE "dy/dx" NOTATION HELPS REMIND US OF THIS.

<u>DEFN</u>. LET $f(x)$ SOME (DIFFERENTIABLE) FUNCTION DEFINED AT THE POINT $x=a$. THE <u>TANGENT LINE</u> OF $f(x)$ AT POINT a IS A LINE $y=mx+b$ WITH $m=f'(a)$ (IT'S A LINE WITH SLOPE $f'(a)$) THAT PASSES THROUGH THE POINT $(a, f(a))$.

CONCEPTUALLY, THIS IS THE LINE WE'LL GET IN THE TEXTBOOK/WINDOW EXAMPLE WHEN WE LET Δt BECOME ARBITRARILY CLOSE TO ZERO.

RECALL.. IF I HAVE A LINE $y=mx+b$, WE SAY IT'S <u>INCREASING</u> WHEN $m>0$, <u>DECREASING</u> WHEN $m<0$, AND <u>CONSTANT</u> WHEN $m=0$.

NOW,.. FOR ANY DIFFERENTIABLE FUNCTION $f(x)$..

- WHENEVER $f'>0$ ON AN INTERVAL (a,b), $f(x)$ IS <u>INCREASING</u> ON (a,b).
- WHENEVER $f'<0$ ON AN INTERVAL (a,b), $f(x)$ IS <u>DECREASING</u> ON (a,b).
- WHENEVER $f'=0$ ON AN INTERVAL (a,b), $f(x)$ IS <u>CONSTANT</u> ON (a,b).

ESSENTIALLY WE CAN THINK OF THE DERIVATIVE $f'(x)$ OF $f(x)$ AS THE SLOPE OF $f(x)$.