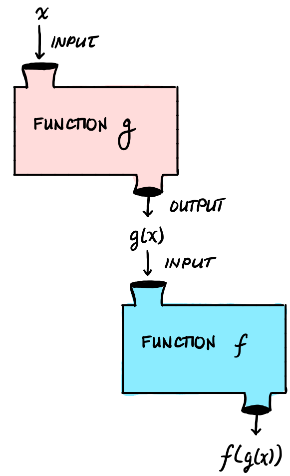
FUNCTION COMPOSITION & THE CHAIN RULE

OBJECTIVES:

- Compute derivatives of products & quotients

FUNCTION COMPOSITION

RECALL: Given functions $f(x)$ and $g(x)$, we can create new functions, the **compositions** of $f(x)$ & $g(x)$: $f(g(x))$ and $g(f(x))$



Ex. Let $f(x) = x^2 + 3x - 1$, $g(x) = e^x$.
Find $f(g(x))$ and $g(f(x))$.

$$
\begin{aligned}
f(g(x)) &= (g(x))^2 + 3(g(x)) - 1 \\
&= (e^x)^2 + 3(e^x) - 1 \\
&= e^{2x} + 3e^x - 1 = f(g(x))
\end{aligned}
$$

$$
\begin{aligned}
g(f(x)) &= e^{(f(x))} \\
&= e^{x^2+3x-1} = g(f(x))
\end{aligned}
$$

ISSUE: How do I compute the derivative of a composition of functions?

IE. $\frac{d}{dx}(x+3)^{50}$?!

DERIVATIVE RULE (CHAIN RULE)

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

Ex. $\frac{d}{dx}(x+3)^{50}$

NOTICE THE FOLLOWING: $f(g(x)) = (x+3)^{50}$

$f(x) = x^{50}$ $\quad f'(x) = 50x^{49}$

$g(x) = x+3$ $\quad g'(x) = 1$

$\to f'(g(x)) \cdot g'(x) = \boxed{50(x+3)^{49} \cdot 1}$

Ex. $\frac{d}{dx}(3x^2-2x)^{17}$

NOTICE: $f(g(x)) = (3x^2-2x)^{17}$

$f(x) = x^{17}$ $\quad f'(x) = 17x^{16}$

$g(x) = 3x^2-2x$ $\quad g'(x) = 6x-2$

$\to f'(g(x)) \cdot g'(x)$

$\boxed{17(3x^2-2x)^{16} \cdot (6x-2)}$

Ex $\frac{d}{dx}(x+2)^2$

CHAIN RULE:

$f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = x+2$ $\quad g'(x) = 1$

$f'(g(x))\, g'(x)$

$2(x+2)(1) = \boxed{2x+4}$

POWER RULE: $\frac{d}{dx}(x+2)(x+2) = \frac{d}{dx}(x^2+4x+4)$

$= \boxed{2x+4}$

THEY'RE THE SAME!

Ex.

- $\frac{d}{dx}\left(e^{3x+7}\right)^{20}$
- $\frac{d}{dx} \frac{1}{x^{12}+2x^{10}-17}$
- $\frac{d}{dx}\sqrt{15x^{12}-7x^3+4}$
- $\frac{d}{dx}\left(\frac{1}{5}x^2+4x\right)^{-4/5}$
- $\frac{d}{dx}\frac{1}{\sqrt[3]{2x^4-3x-1}}$
- $\frac{d}{dx} e^{5x}$
- $\frac{d}{dx} 5^{3x^2-7x+\frac{1}{x}}$
- $\frac{d}{dx}\frac{e^{2x}}{x}$
- $\frac{d}{dx} e^{3x^2+2x-1}$