DERIVATIVES OF PRODUCTS & QUOTIENTS

OBJECTIVES:

- COMPUTE DERIVATIVES OF PRODUCTS & QUOTIENTS

ISSUE: (FROM LAST CLASS) HOW CAN I COMPUTE $\frac{d}{dx} e^x(x^2+1)$?!

OK.. LETS TRY DISTRIBUTING!

$$\frac{d}{dx} e^x(x^2+1) = \frac{d}{dx} e^x x^2 + \frac{d}{dx} e^x$$

WE CAN FIND THIS!

THIS IS THE PROBLEM.

SO WHAT CAN WE DO?!

YOU CAN CHECK THAT $\frac{d}{dx} e^x x^2 \neq \left(\frac{d}{dx} e^x\right)\left(\frac{d}{dx} x^2\right)$

DERIVATIVE RULE (PRODUCT RULE)

$$\left[f(x)g(x)\right]' = f'(x)g(x) + f(x)g'(x) \quad \text{EQUIV:} \quad \frac{d}{dx}\left(f(x)g(x)\right) = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$$

EX TO CONVINCE YOU THAT THIS IS REASONABLE.. $\frac{d}{dx}(x+2)(x-1)$

PRODUCT RULE:

$\frac{d}{dx}\underbrace{(x+2)}_{f(x)}\underbrace{(x-1)}_{g(x)}$

$f(x) = x+2 \quad f'(x) = 1$

$g(x) = x-1 \quad g'(x) = 1$

PLUGGING INTO PRODUCT RULE...

$= f'(x)g(x) + f(x)g'(x)$

$= 1\cdot(x-1) + (x+2)\cdot 1$

$= x-1+x+2$

$= \boxed{2x+1}$

POWER RULE

$\frac{d}{dx}(x+2)(x-1)$ FOIL

$= \frac{d}{dx} x^2 - x + 2x - 2$

$= \frac{d}{dx} x^2 + x - 2$

$= \boxed{2x+1}$

NOTICE THAT WE GET THE SAME SOLUTION...

EX. $\frac{d}{dx}\underbrace{e^x}_{f(x)}\underbrace{(x^2+1)}_{g(x)}$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = x^2+1 \quad g'(x) = 2x$

PLUG INTO PRODUCT RULE

$f'(x)g(x) + f(x)g'(x)$

$e^x(x^2+1) + 2xe^x$

SO OUR FINAL ANSWER IS:

$$\boxed{e^x(x^2+1) + 2xe^x}$$

Ex

- $\frac{d}{dx}\left(x^2+7x-3\right)\left(x^{60}-5x^{27}\right)$
- $\frac{d}{dx}\sqrt[3]{x^2}\left(2x-x^2\right)$ → WE DID THIS A DIFFERENT WAY LAST CLASS
- $\frac{d}{dx}\ e^x \cdot e^x$
- $\frac{d}{dx}$

ISSUE: HOW CAN I COMPUTE $\frac{d}{dx}\ \frac{x^7-6x^4+3}{2e^x-7}$?!

DERIVATIVE RULE (QUOTIENT RULE)

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)\,g(x)-f(x)\,g'(x)}{(g(x))^2}$$

EQUIVALENTLY... WE CAN WRITE USING d/dx NOTATION.

Ex $\frac{d}{dx}\ \frac{x^7-6x^4+3}{2e^x-7}$ → $f(x)$ = TOP; → $g(x)$ = BOTTOM

$f(x) = x^7-6x^4+3 \longrightarrow f'(x) = 7x^6-24x^3$

$g(x) = 2e^x-7 \longrightarrow g'(x) = 2e^x$

APPLY QUOTIENT RULE...

$$\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2} = \frac{(7x^6-24x^3)(2e^x-7)-(x^7-6x^4+3)(2e^x)}{(2e^x-7)^2}$$

FINAL ANSWER