DERIVATIVES

FROM LAST CLASS, WE FOUND...

- DERIVATIVE RULE $\frac{d}{dx}(x^n) = nx^{n-1}$ WHENEVER $n \neq 0$. (POWER RULE)

PROPERTIES OF DERIVATIVES

TO DIFFERENTIATE SUMS/DIFFERENCES, YOU CAN DIFFERENTIATE THE INDIVIDUAL TERMS

(i) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ OR YOU CAN WRITE $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(ii) $(c\,f(x))' = c \cdot f'(x)$ FOR A CONSTANT c; OR YOU CAN WRITE: $\frac{d}{dx}(c\,f(x)) = c \cdot \frac{d}{dx}(f(x))$

CONSTANTS HOP OUT FRONT OF THE DERIV.

EXS DIFFERENTIATE THE FOLLOWING

(A) $f(x) = x^{5001}$

$(f'(x) = nx^{n-1})$

POWER RULE: $n = 5001$ $\longrightarrow$ $f'(x) = 5001\,x^{5000}$

(B) $f(x) = 5$

DERIV. OF CONSTANT IS ZERO. $\longrightarrow$ $f'(x) = 0$

(C) $g(t) = 2t^6 + 7t^{-6}$

sum of 2 guys!

DEAL WITH THEM SEPARATELY

$2t^6 \quad + \quad 7t^{-6}$

$\frac{d}{dx}(2t^6) = 2 \cdot \frac{d}{dx}(t^6)$

$= 2(6t^5)$

$= 12t^5$

$\frac{d}{dx}(7t^{-6}) = 7\frac{d}{dx}(t^{-6})$

$= 7(-6t^{-7})$

$= -42t^{-7}$

CAREFUL HERE...

$-6 - 1 = -7$

FINAL ANSWER: $\boxed{12t^5 - 42t^{-7}}$

(D) $T(z) = \sqrt{z} + 9\sqrt[3]{z^7} - \frac{2}{\sqrt[5]{z^2}}$

WE CAN USE THE POWER RULE FOR THIS IF WE JUST REWRITE THE EXPONENTS!

REWRITE $z^{1/2} \quad 9(z^7)^{1/3} - 2(z^2)^{-1/5}$

$z^{1/2} + 9z^{7/3} - 2z^{-2/5}$ $\longrightarrow$ NOW TAKE DERIVATIVE OF THIS

$\frac{d}{dz}(z^{1/2}) + 9\frac{d}{dz}(z^{7/3}) - 2\frac{d}{dz}(z^{-2/5})$

$$\frac{d}{dz}\left(z^{1/2}\right) + 9\frac{d}{dz}\left(z^{7/3}\right) - 2\frac{d}{dz}\left(z^{-2/5}\right)$$

$\frac{d}{dz}\left(z^{1/2}\right)$ $n = 1/2$

$n-1 = -1/2$

$= 1/2\, z^{-1/2}$

$n = 7/3$

$n-1 = \frac{7}{3} - \frac{3}{3} = \frac{4}{3}$

$= 9\left(7/3\, z^{4/3}\right)$

$n = \frac{-2}{5}$

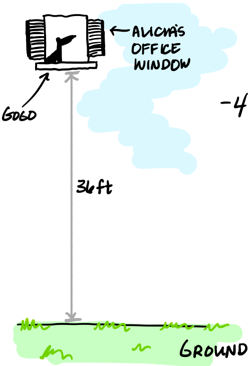
$n-1 = -\frac{2}{5} - \frac{5}{5} = \frac{-7}{5}$

$= -2\left(-2/5\, z^{-7/5}\right)$

SO OUR FINAL SOLUTION SHOULD BE...

$$\boxed{\frac{1}{2}z^{-1/2} + 21z^{4/3} + \frac{4}{5}z^{-7/5}}$$

Ex RECALL THE WINDOW/TEXTBOOK PROBLEM.



THE SITUATION WAS MODELED BY $-t^2+2t+36$.

WE WANTED TO CHECK IF ALICIA'S CLAIM THAT THE VELOCITY AT $t=4$ is -4 ft/sec WAS TRUE.

$\frac{d}{dt}\left(-t^2+2t+36\right)$ GIVES THE SLOPE FOR $-t^2+2t+36$.

$$\frac{d}{dt}\left(-t^2+2t+36\right) = \frac{d}{dt}\left(-t^2\right) + \frac{d}{dt}(2t) + \frac{d}{dt}(36)$$

$$= -2t + 2$$

DERIVATIVE IS $\underline{-2t+2}$

TO FIND DERIVATIVE @ $t=4$, WE EVALUATE $-2t+2$ AT $t=4$.

$$-2t+2\Big|_{t=4} = -2(4)+2$$
$$= -8+2 = \boxed{-6 \text{ FT/SEC}}$$

WHICH WE FOUND ON WEDNESDAY!

IS THERE ANY TIME t FOR WHICH THE VELOCITY WAS -4 ft/sec?

IS THERE ANY TIME t WHEN THE VELOCITY IS 0 FT/SEC?

INTERPRET WHAT THIS MEANS..

On Friday we had some problems with tangent lines..

Ex. Find the line tangent to $-x^2+2x+36$ at $x=4$.

<u>Want:</u> line with slope -6 passing through $(4, -16+8+36)$

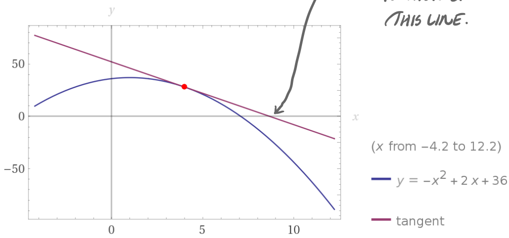
$$= (4, 28)$$

We found the equation of this line.

$y = mx + b \rightsquigarrow y = -6x + b$

To solve for b, we can plug in $(4,28)$.

(Check that you get $\underline{y = -6x+52}$.)



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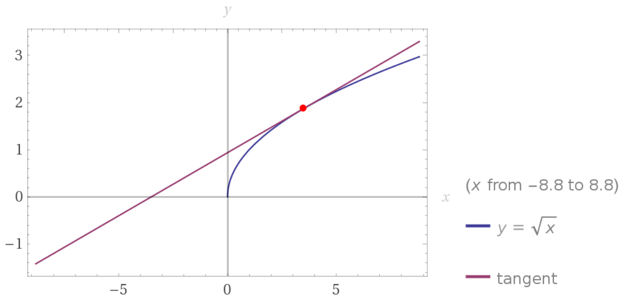
Ex. Find the line tangent to $\sqrt{x}$ at $x=3.5$.



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