Derivatives

Definition and Notation

If $y = f(x)$ then the derivative is defined to be $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

If $y = f(x)$ then all of the following are equivalent notations for the derivative.

$$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}\left(f(x)\right) = Df(x)$$

If $y = f(x)$ all of the following are equivalent notations for derivative evaluated at $x = a$.

$$f'(a) = y'\big|_{x=a} = \left.\frac{df}{dx}\right|_{x=a} = \left.\frac{dy}{dx}\right|_{x=a} = Df(a)$$

Interpretation of the Derivative

If $y = f(x)$ then,

1. $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$ and the equation of the tangent line at $x = a$ is given by $y = f(a) + f'(a)(x - a)$.
2. $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.
3. If $f(x)$ is the position of an object at time x then $f'(a)$ is the velocity of the object at $x = a$.

Basic Properties and Formulas

If $f(x)$ and $g(x)$ are differentiable functions (the derivative exists), c and n are any real numbers,

1. $(c\,f)' = c\,f'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(f\,g)' = f'\,g + f\,g'$ – **Product Rule**
4. $\left(\dfrac{f}{g}\right)' = \dfrac{f'\,g - f\,g'}{g^2}$ – **Quotient Rule**
5. $\dfrac{d}{dx}(c) = 0$
6. $\dfrac{d}{dx}(x^n) = n\,x^{n-1}$ – **Power Rule**
7. $\dfrac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$

   This is the **Chain Rule**

Common Derivatives

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln(a)$$

$$\frac{d}{dx}\left(\mathbf{e}^x\right) = \mathbf{e}^x$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x},\quad x > 0$$

$$\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x},\quad x \neq 0$$

$$\frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{x \ln a},\quad x > 0$$