Sols

Math 122: Integration by Substitution Practice

For each problem, identify what (if any) u-substitution needs to be made to evaluate each integral. Make the substitution, simplify, evaluate the integral, and, for indefinite integrals, remember to write your answer in terms of the original variable.

1. $\int x^2 e^{x^3+1} dx$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{1}{3}\int e^u du$

$= \frac{1}{3}e^u + C$

$= \frac{1}{3}e^{x^3+1} + C$

2. $\int 2x(x^2+1)^5 dx$

$u = x^2 + 1$

$du = 2x\,dx$

$\int u^5 du$

$= \frac{u^6}{6} + C = \frac{(x^2+1)^6}{6} + C$

3. $\int (5x+1)^9 dx$

$u = 5x + 1$

$du = 5\,dx$

$\frac{1}{5}\int u^9 du$

$= \frac{1}{5} \cdot \frac{u^{10}}{10} + C = \frac{(5x+1)^{10}}{50} + C$

4. $\int \frac{t}{1+3t^2} dt$

$u = 1 + 3t^2$

$du = 6t\,dt$

$\frac{1}{6}\int \frac{1}{u} du$

$= \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|1+3t^2| + C$

5. $\int x\sqrt{3x^2+4}\,dx$

$u = 3x^2 + 4$

$du = 6x\,dx$

$\frac{1}{6}\int u^{1/2} du$

$= \frac{1}{6} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{9}(3x^2+4)^{3/2} + C$

6. $\int \frac{x+1}{x^2+2x+9}dx$

$u = x^2+2x+9$

$du = 2x+2\,dx = 2(x+1)dx$

$\frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x+9| + C$

7. $\int \frac{e^t+1}{e^t+t}dt$

$u = e^t + t$

$du = e^t + 1\,dt$

$\int \frac{1}{u}du$

$= \ln|u| + C$

$= \ln|e^t+t| + C$

8. $\int \frac{1}{\sqrt{t+1}}dt$

$u = t+1$

$du = 1dt$

$\int \frac{1}{\sqrt{u}}du$

$= \int \frac{1}{u^{1/2}}du = \int u^{-1/2}du = 2u^{1/2} + C = 2(t+1)^{1/2} + C$

9. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}dx$

$2\int e^u du$

$= 2e^u + C = 2e^{\sqrt{x}} + C$

10. $\int \frac{2x}{x^2+1}dx$

$u = x^2+1$

$du = 2x\,dx$

$\int \frac{1}{u}dx$

$= \ln|u| + C$

$= \ln|x^2+1| + C$

* For substitution with DEFINITE integrals, you have two options:

Method 1

11. $\int_0^2 x(x^2+1)^2 dx$

$\frac{1}{2}\int_{D}^{D} u^2 du = \frac{1}{2}\cdot\frac{u^3}{3}\Big|_D^D = \frac{(x^2+1)^3}{6}\Big|_0^2$

$u = x^2+1$

$du = 2x dx$

$= \frac{(2^2+1)^3}{6} - \frac{(0^2+1)^3}{6}$

$= \frac{125}{6} - \frac{1}{6} = \frac{124}{6} = \frac{62}{3}$

Method 1

12. $\int_0^1 2te^{-t^2} dt$

$-\int_D^D e^u du = -e^u\Big|_D^D$

$u = -t^2$

$du = -2t dt$

$= -e^{-t^2}\Big|_0^1 = -e^{-1^2} - -e^{-0^2} = -e^{-1} + 1$

Method 2

13. $\int_1^3 \frac{1}{(t+7)^2} dt$

$\int_8^{10} \frac{1}{u^2} du = \int_8^{10} u^{-2} du$

$u = t+7$ $t=3$ $u=10$

$du = 1dt$ $t=1$ $u=8$

$= -u^{-1}\Big|_8^{10} = -\frac{1}{10} + \frac{1}{8} = \frac{1}{40}$

Method 2

14. $\int_0^2 \frac{x}{(1+x^2)^2} dx$

$\frac{1}{2}\int_1^5 \frac{1}{u^2} du = \frac{1}{2}\int_1^5 u^{-2} du$

$u = 1+x^2$ $x=2$ $u=5$

$x=0$ $u=1$

$du = 2x dx$

$= -\frac{1}{2}u^{-1}\Big|_1^5 = -\frac{1}{2}\cdot\frac{1}{5} + \frac{1}{2}\cdot\frac{1}{1}$

$= -\frac{1}{10} + \frac{1}{2} = \frac{2}{5}$

Method 2

15. $\int_{-1}^{e-2} \frac{1}{t+2} dt$

$\int_1^e \frac{1}{u} du = \ln|u|\Big|_1^e$

$= \ln|e| - \ln|1|$

$u = t+2$ $t = e-2$ $u = e$

$du = 1dt$ $t=-1$ $u=1$

$= 1 - 0$

$= 1$