Solutions

Calculus I **Project: Solar energy, accumulation, and Riemann sums**

The graph on the last page of this project depicts power generation and power consumption at an elementary school in Boulder. The vaguely bell-shaped curve (the green curve on the color graph), call it $g(t)$, represents power **generated** from solar panels installed at the school. The relatively level, jagged curve (the red curve on the color graph), call it $c(t)$, represents power **consumption** at the school.

The vertical axis is the power axis. The range is 0-10 kilowatts (kW). The horizontal axis is the time axis. The domain is from 12 AM Monday, March 18 until 12 AM Tuesday, March 19, 2013. Each of the ticks on the time axis represents 1/2 hour. (The bolder ticks are spaced 3 hours apart.)

To complete this project, you'll need to know that power times time equals energy (assuming power is supplied at a constant rate over the time interval in question).

1. Consider the bell-shaped (green) curve $g(t)$. What quantity does the area under this curve, between two points $t = a$ and $t = b$, represent? What are the units for this quantity?

   $g(t)$ represents power generated in kW. Since the horizontal axis is time (in hours), the area under $g(t)$ is total energy generated in kilowatt hours.

2. Consider the relatively flat (red) curve $c(t)$. What quantity does the area under this curve, between two points $t = a$ and $t = b$, represent? What are the units for this quantity?

   $c(t)$ represents power consumption in kW.
   The area under $c(t)$ is total energy consumed in kWhours.

3. On the graph, draw right endpoint Riemann sums, of baselength $\Delta t = 2$ hour, representing approximations to:

   (a) The cumulative energy, call it $G(T)$, *generated* between $t = 0$ and $t = T$ (where $t = 0$ is 12 AM Monday, March 18, 2013);

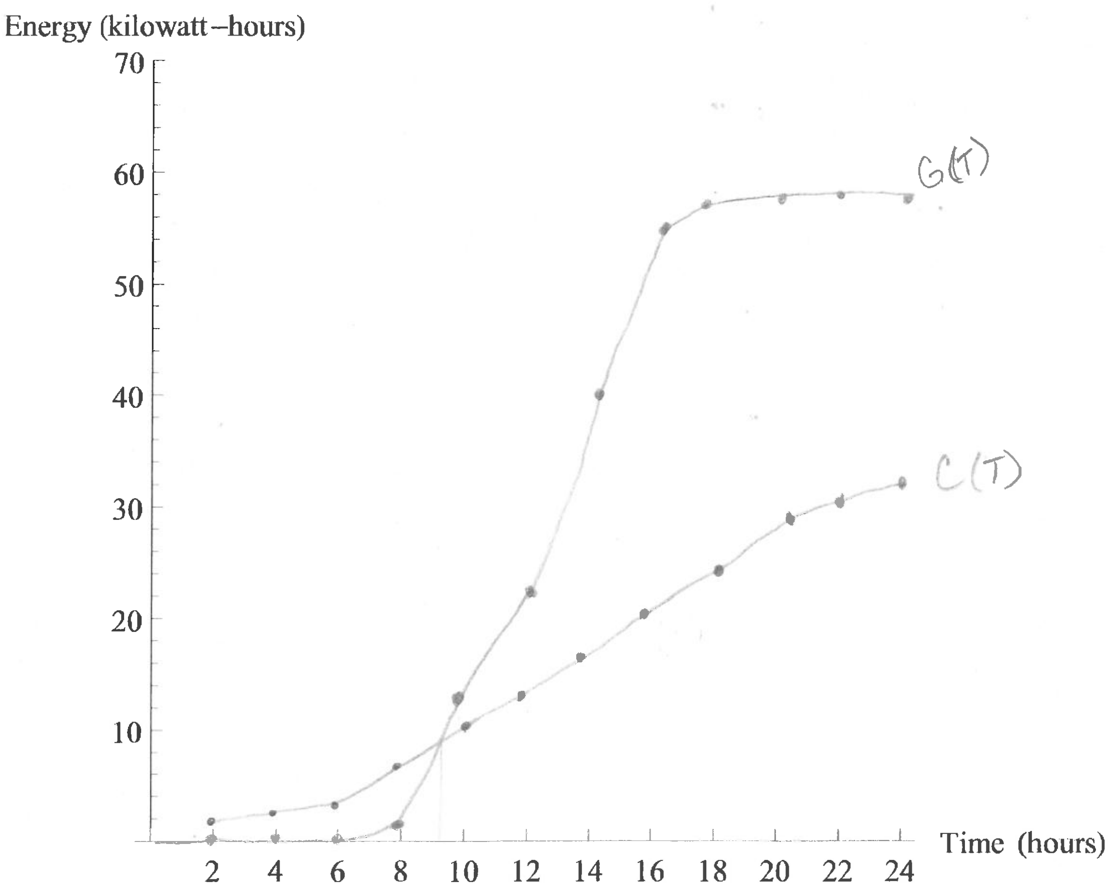
   (b) The cumulative energy, call it $C(T)$, *consumed* between $t = 0$ and $t = T$.

   See graph

4. Using the Riemann sums you drew for the previous exercise, fill out the table, below, of (approximate) values of $g(t)$, $G(T)$, $c(t)$ and $C(T)$.

T	2	4	6	8	10	12	14	16	18	20	22	24
$g(T)$	0	0	0	.4	6.5	8.9	9	6.8	.4	0	0	0
$G(T)$	0	0	0	.8	13.8	22.7	40.7	54.3	55.1	55.1	55.1	55.1
$c(T)$	.4	.3	.4	2.1	2.2	2	1.9	1.8	1.6	1.6	1.2	1
$C(T)$	.8	1.4	2.2	6.4	10.8	14.8	18.6	22.2	25.4	28.6	31	33

5. On the axes below, sketch the graphs of $G(T)$ and $C(T)$.



6. Do your above graphs of $G(T)$ and $C(T)$ intersect? If so, where? Which graph ends up higher than the other, and what's the significance of this?

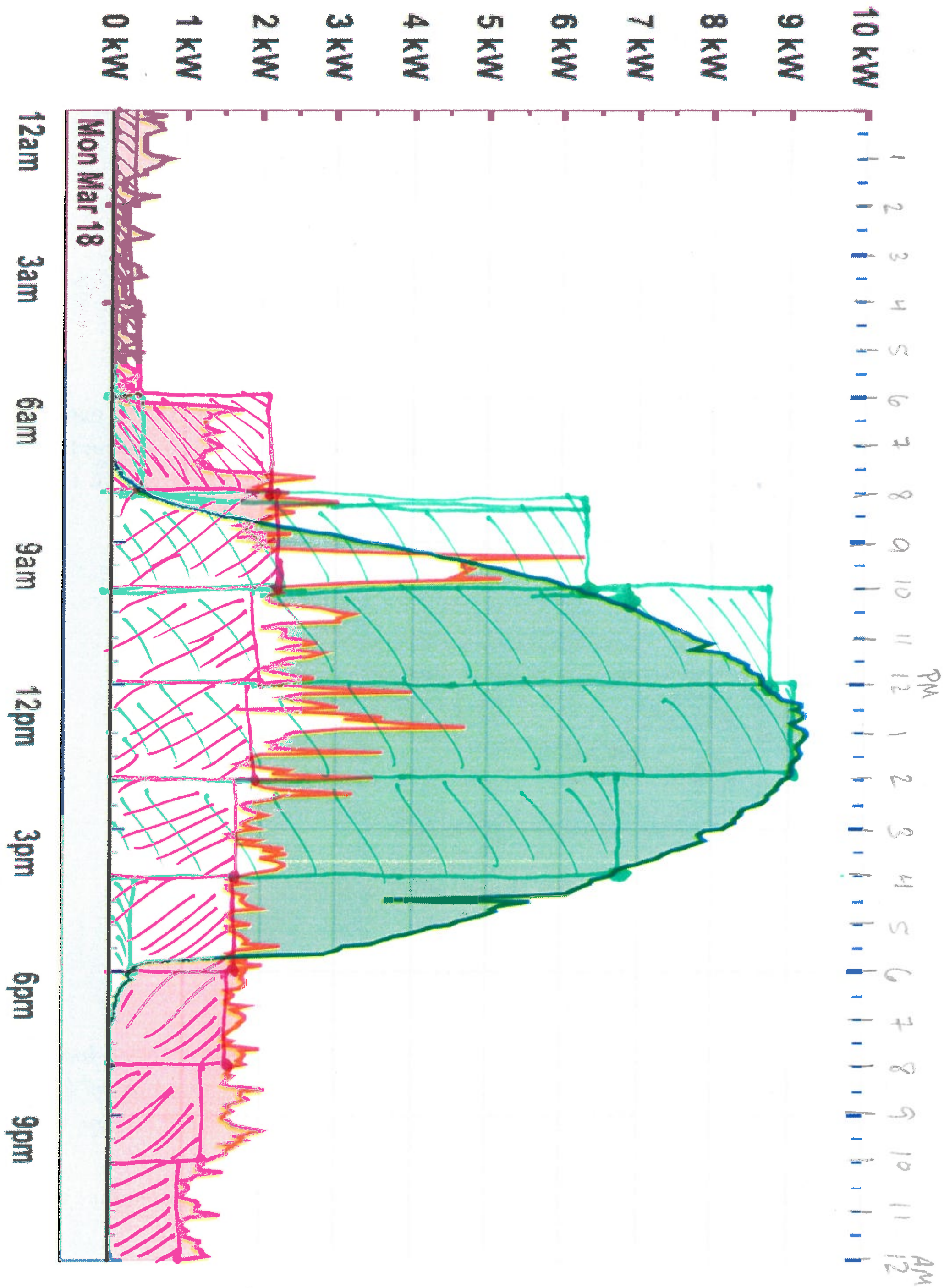
   G(T) and C(T) intersect at $T \approx 9$
   G(T) ends up higher meaning there was more energy generated than consumed

7. If you were to sketch the *derivative* of the function $G(T)$ you sketched in exercise 5 above, what would this derivative look like (very roughly)? Please explain. (You don't actually have to sketch this derivative to answer; in fact, you've already seen the graph of this derivative, very recently!) Answer the same question for your graph of $C(T)$.

   It would look like g(t)!
   The graph of the derivative of C(T) would look like c(T)!
   G(T) and C(T) are the respective antiderivatives (integrals) of g(T) and c(T).

8. Fill in the blanks: The function $G(T)$ above, representing energy generated, has something of an elongated "S" shape. On the other hand, the graph $g(t)$ of generated *power* (see, again, the graph on the last page), which is the <u>derivative</u> of generated energy, has something of a <u>bell</u> shape.

   (That the derivative of an "S" curve is a bell curve is a general phenomenon, which you will perhaps encounter elsewhere in this course, and beyond.)


0 kW
1 kW
2 kW
3 kW
4 kW
5 kW
6 kW
7 kW
8 kW
9 kW
10 kW
Mon Mar 18
12am
3am
6am
9am
12pm
3pm
6pm
9pm
PM
AM