Math 122 Calculus for Business Admin. and Social Sciences

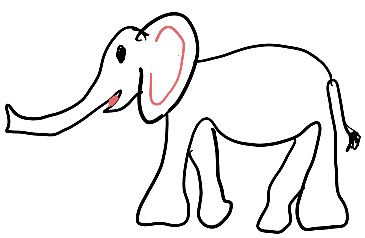
Exam #3 A

July 31, 2018

Instructor: Ann Clifton

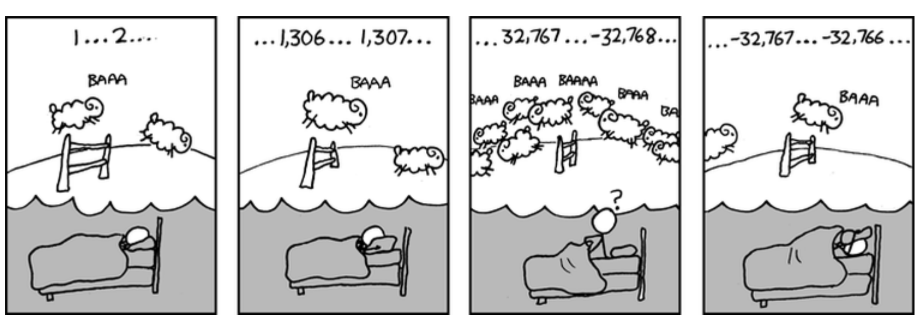
Name: Sols

Do not turn this page until told to do so. You will have a total of 1 hour 40 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers will **not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw an elephant on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.



#	score	out of	#	score	out of
1		3	9		6
2		4	10		20
3		3	11		16
4		3	12		15
5		3	13		5
6		4	14		10
7		4	EC		5
8		4	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



Fill in the blanks.

1. (3 points) (Fundamental Theorem of Calculus) If $f(x)$ is a continuous function on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = \underline{F(b) - F(a)}$$

2. (4 points) Assume that $\int f(x)dx$ and $\int g(x)dx$ exist.

   (a) $\int f(x) \pm g(x)dx = \underline{\int f(x)dx \pm \int g(x)\,dx}$

   (b) Let a be a number, $\int af(x)dx = \underline{a\int f(x)dx}$

3. (3 points) Let $n \neq -1$ be a fixed number,

$$\int x^n dx = \underline{\frac{x^{n+1}}{n+1} + c}$$

4. (3 points)

$$\int e^x dx = \underline{e^x + c}$$

5. (3 points)

$$\int \frac{1}{x} dx = \underline{\ln|x| + c}$$

Multiple Choice. Choose the best answer. (4 points each.)

6. Find the antiderivative $F(x)$ of the function $f(x) = 3x^2 + e^x$ which satisfies $F(0) = 2$.

B

A. $F(x) = x^3 + e^x + 2$ **B.** $F(x) = x^3 + e^x + 1$

C. $F(x) = x^3 + e^x + c$ **D.** $F(x) = x^3 + e^x + 3$

$F(x) = x^3 + e^x + c$

$F(0) = 0 + 1 + c = 2$

$c = 1$

7. Find the indefinite integral $\int \left(\frac{3}{x} + \frac{1}{\sqrt{x}}\right) dx$.

C

A. $2\sqrt{x} + c$ **B.** $3 \ln x + \frac{2}{\sqrt{x}} + c$

C. $3 \ln |x| + 2\sqrt{x} + c$ **D.** $3 \ln |x| + \frac{2}{\sqrt{x}} + c$

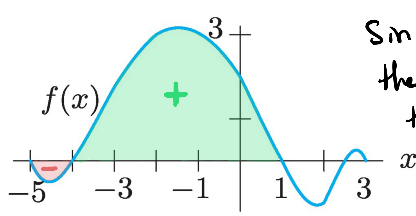
$\int 3 \cdot \frac{1}{x} + x^{-1/2}\, dx$

$= 3 \ln|x| + \frac{x^{1/2}}{1/2} + C$

$= 3 \ln|x| + 2x^{1/2} + C$

8. Using the graph below, determine whether $\int_{-5}^{1} f(x)dx$ is positive, negative, approximately zero, or if there is not enough information.

A



Since the green area above the x-axis is clearly larger than the red area below the x-axis, the net result is positive.

A. Positive **B.** Negative

C. Approximately Zero **D.** Not enough information

Short Answer.

9. (6 points) Approximate the area under the curve $y = x^2$ on the interval $[0, 4]$ using $n = 4$ right-endpoint subintervals.

$f(x) = x^2$, $n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$f(1) = 1$

$f(2) = 4$

$f(3) = 9$

$f(4) = 16$

$\Delta x(f(1) + f(2) + f(3) + f(4))$

$= 1(1 + 4 + 9 + 16)$

$= \boxed{30}$

10. (20 points) Compute the following indefinite integrals.

(a) $\int 7dx$

$7x + C$

(b) $\int (10x + 2)dx$

$5x^2 + 2x + C$

(c) $\int (36x^2 + 26x)dx$

$12x^3 + 13x^2 + C$

(d) $\int x^2 dx$

$\frac{x^3}{3} + C$

(e) $\int \frac{1}{\sqrt{x}}dx$ $= \int x^{-1/2} dx = 2x^{1/2} + C$

11. (16 points) Compute the following indefinite integrals.

(a) $\int 25(x+7)^{24}dx$

$(x+7)^{25} + C$

(b) $\int (x+2)e^{\frac{1}{2}x^2+2x+1}dx$

$u = \frac{1}{2}x^2 + 2x + 1$

$du = x + 2\ dx$

$\int e^u\, du$

$= e^u + C$

$= e^{\frac{1}{2}x^2 + 2x + 1} + C$

(c) $\int \frac{4x}{2x^2+7}dx$

$u = 2x^2 + 7$

$du = 4x\, dx$

$\int \frac{1}{u}\, du$

$= \ln|u| + C$

$= \ln|2x^2 + 7| + C$

(d) $\int \frac{x}{\sqrt{x^2+1}}dx$

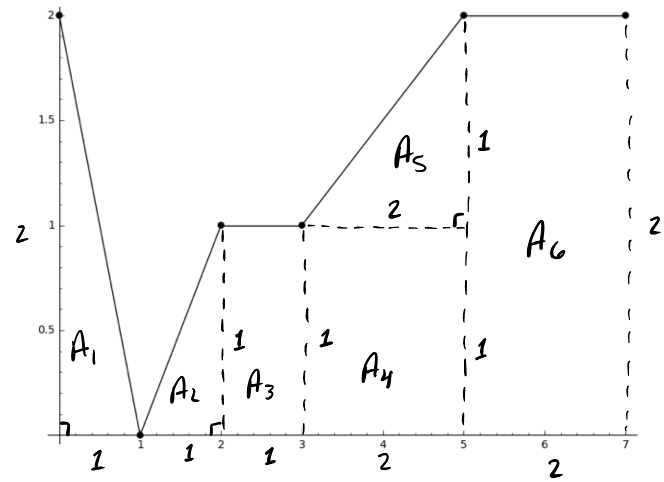
$u = x^2 + 1$

$du = 2x\, dx$

$\frac{1}{2}\int \frac{1}{\sqrt{u}}\, du = u^{1/2} + C$

$= (x^2+1)^{1/2} + C$

12. (15 points) Consider the function f given by the graph:



Compute $\int_0^7 f(x)dx$.

$$\int_0^7 f(x)dx = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \left(= \sum_{i=1}^{6} A_i\right)$$

$$= \frac{1}{2}(1)(2) + \frac{1}{2}(1)(1) + 1(1) + 2(1) + \frac{1}{2}(2)(1) + 2(2)$$

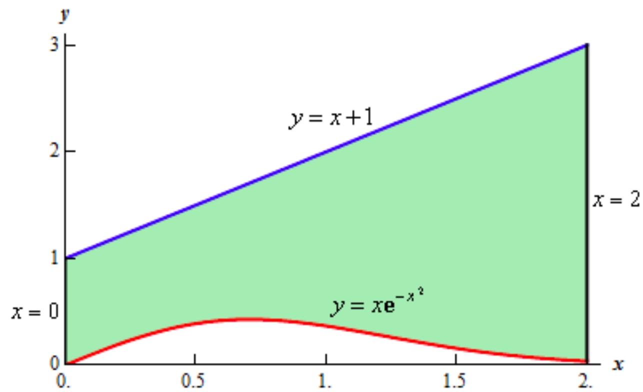
$$= 1 + \frac{1}{2} + 1 + 2 + 1 + 4$$

$$= 9\frac{1}{2}$$

13. (5 points) What is your favorite color?

Green

14. (10 points) Find the area of the region bounded by $y = xe^{-x^2}$ and $y = x + 1$ on the interval $[0, 2]$. *Set up but do* **not** *evaluate the integral.* The graph of the region is given below for reference.



$$A = \int_0^2 (x+1) - (xe^{-x^2})\,dx$$

15. (Extra Credit 5 points) Evaluate the integral from number 14 (the problem above). Round your answer to four decimal places.

$$A = \int_0^2 (x+1)dx - \int_0^2 xe^{-x^2}dx \qquad u = -x^2 \qquad du = -2x\,dx \qquad x=0 \quad u=0 \qquad x=2 \quad u=-4$$

$$= \left[\frac{x^2}{2} + x\right]_0^2 - \left(-\frac{1}{2}\int_0^{-4} e^u du\right) = \left[\frac{4}{2} + 2 - 0\right] + \frac{1}{2} e^u \Big|_0^{-4}$$

$$= 4 + \frac{1}{2}(e^{-4} - 1) \approx 3.5092$$