Math 122 Calculus for Business Admin. and Social Sciences — Exam #2 A

July 18, 2018

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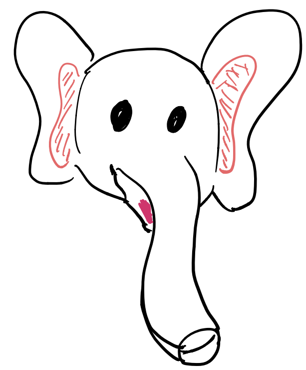
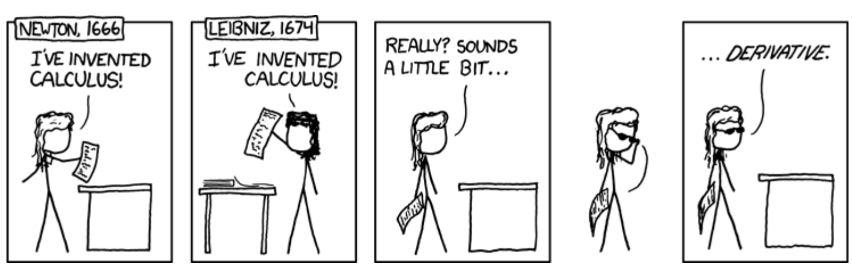
Do not turn this page until told to do so. You will have a total of 1 hour 40 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers will **not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw an elephant on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.



#	score	out of	#	score	out of
1		3	9		10
2		3	10		12
3		9	11		12
4		10	12		10
5		4	13		16
6		4	EC		5
7		2	Total		100
8		5			

Remember: This exam has no impact on your worth as a human being. You got this!!!



Throughout this section, let f and g be differentiable functions. Fill in the blanks.

1. (3 points)

   (a) Let a be a constant; $\frac{d}{dx}(af(x)) =$ $af'(x)$

   (b) $\frac{d}{dx}(f(x) + g(x)) =$ $f'(x) + g'(x)$

   (c) $\frac{d}{dx}(f(x) - g(x)) =$ $f'(x) - g'(x)$

2. (3 points)

   (a) For n a number, $\frac{d}{dx}(x^n) =$ nx^{n-1}

   (b) $\frac{d}{dx}(\ln(x)) =$ $1/x$

   (c) $\frac{d}{dx}(e^x) =$ e^x

3. (9 points) Write the formula for each of the following derivatives.

(a)

$$\frac{d}{dx}(f(x)g(x))$$

$$f'(x)g(x) + f(x)g'(x)$$

(b)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

(c)

$$\frac{d}{dx}(f \circ g(x))$$

$$f'(g(x))g'(x)$$

Multiple Choice For each of the following questions, circle the correct answer.

4. (10 points) Assume that f is a function such that $f'(x)$ and $f''(x)$ are defined for all x.

A

(a) A point p is a critical point of f if

A. $f'(p) = 0$ **B.** $f'(p) < 0$

C. $f'(p) > 0$ **D.** $f(p) = 0$

(b) f is increasing on an interval if

B

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

(c) f is decreasing on an interval if

A

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

(d) f is concave down on an interval if

C

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

(e) f is concave up on an interval if

B

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

5. (4 points) The **first** derivative test says that a critical point, p, of f is a

(a) local maximum if

B

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

(b) local minimum if

A

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

6. (4 points) The **second** derivative test says that a critical point, p, of f is a

(a) local maximum if

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C

C. $f''(p) < 0$

D. $f''(p) > 0$

(b) local minimum if

D

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C. $f''(p) < 0$

D. $f''(p) > 0$

7. (2 points) Suppose that $f''(p) = 0$. We say that p is an inflection point of f if

A

A. f'' changes sign at p

B. f changes sign at p

C. $f'(p) = 0$

D. $f(p) = 0$

8. (5 points) Find the derivative of the following functions.

(a) $f(x) = 3x + 7$

3

(b) $g(x) = 5x^2 + 2x + 1$

$10x + 2$

(c) $h(x) = 12x^3 + 13x^2$

$36x^2 + 26x$

(d) $r(x) = \frac{1}{3}x^3 + 2$

x^2

(e) $s(x) = \sqrt{x} + 3$

$\frac{1}{2}x^{-1/2}$

9. (10 points) Find the derivative of the following functions.

(a) $(x+7)^{25}$

$$25(x+7)^{24}$$

(b) $e^{\frac{1}{2}x^2+2x+1}$

$$(x+2)e^{\frac{1}{2}x^2+2x+1}$$

(c) $\ln(2x^2+7)$

$$\frac{4x}{2x^2+7}$$

(d) $\sqrt{x^2+1}$

$$\frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2}$$

(e) $6e^{5x}+e^{-x^2}$

$$30e^{5x}-2xe^{-x^2}$$

10. (12 points) Differentiate the following functions.

(a) xe^{-2x}

$e^{-2x} - 2xe^{-2x}$

(b) $x\ln(x)$

$\ln(x) + 1$

(c) $(x^2 + 3)e^x$

$2xe^{x} + (x^2+3)e^{x}$

11. (12 points) Differentiate the following functions.

(a) $\dfrac{x+1}{x-1}$

$$\frac{x-1-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

(b) $\dfrac{x}{e^x}$

$$\frac{e^x - xe^x}{(e^x)^2} = \frac{1-x}{e^x}$$

(c) $\dfrac{x}{\ln(x)}$

$$\frac{\ln(x)-1}{(\ln(x))^2}$$

12. (10 Points) Let $f(x) = 10x^4 - 4x^5$.

(a) Find the derivative of f.

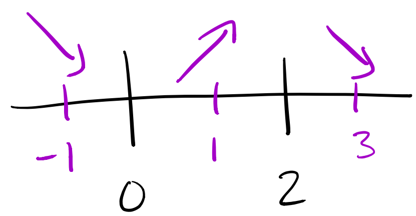
$$f'(x) = 40x^3 - 20x^4 = 20x^3(2-x)$$

(b) Find the critical points of f. [Hint: Factoring after taking the derivative will make this much easier.]

$$20x^3(2-x) = 0$$

$$x = 0, \quad x = 2$$

(c) Find any local maxima and local minima of f. Clearly indicate whether a point is a maximum or a minimum.



$f'(-1) < 0$ $f'(1) > 0$ $f'(3) < 0$

$x = 0$ Local min

$x = 2$ Local max

(d) Find the global maximum and global minimum on the interval $[1, 3]$.

$f(1) = 10(1)^4 - 4(1)^5 = 6$

$f(0) = 10(0)^4 - 4(0)^5 = 0$

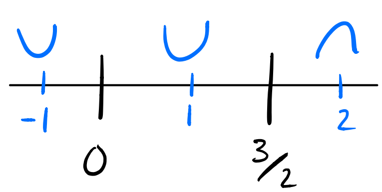
$\boxed{f(2) = 10(2)^4 - 4(2)^5 = 32}$ Global max

$\boxed{f(3) = 10(3)^4 - 4(3)^5 = -162}$ Global min

(e) Find any inflection points of f.

$$f''(x) = 120x^2 - 80x^3 = 40x^2(3-2x) = 0$$

$x = 0$ $\boxed{x = 3/2}$ Inflection Point



$f''(-1) > 0$ $f''(1) > 0$ $f''(2) < 0$

13. (16 points) A company sells a product for \$30 each and the manufacturing costs can be modeled by the function

$$C(q) = q^3 - 9q^2 + 45q + 15$$

of q units produced. For each of the quantities below, determine whether the company should increase, decrease, or not change the production levels in order to maximize profit. Justify your answers using calculus. **You will not receive credit for guess and check solutions.**

[Hint: Find the quantity that maximizes the profit function.]

(a) $q = 2$ Increase (b) $q = 6$ Decrease (c) $q = 7$ Decrease

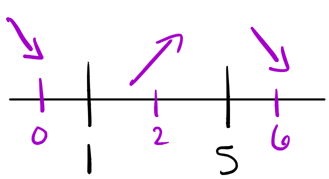
$R(q) = 30q$

$\pi(q) = R(q) - C(q) = 30q - (q^3 - 9q^2 + 45q + 15)$

$= -q^3 + 9q^2 - 15q - 15$

$\pi'(q) = -3q^2 + 18q - 15 = -3(q^2 - 6q + 5) = -3(q-5)(q-1) = 0$

CPs $q=5$ $q=1$



$\pi'(0) < 0$

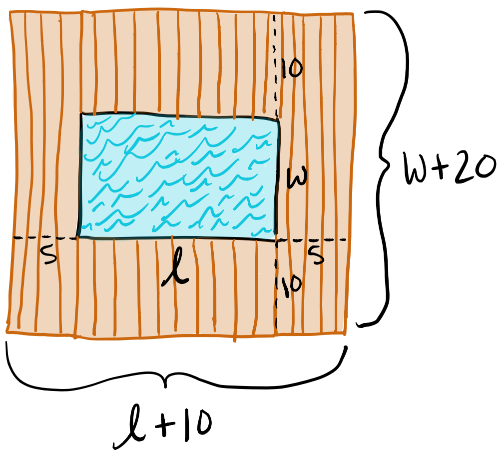
$\pi'(2) > 0$

$\pi'(6) < 0$

$q = 5$ is the quantity that maximizes profit. So, for any production level less than 5 the company should increase and for any greater than 5, the company should decrease

Extra Credit No partial credit will be given for this problem.

A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot-wide decks along either side and 10-foot-wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.



$$A_{pool} = l \cdot w = 1800 \Rightarrow l = \frac{1800}{w}$$

$$A = (l+10)(w+20)$$

$$A = lw + 20l + 10w + 200$$

$$A = \frac{1800}{w} \cdot w + 20 \cdot \frac{1800}{w} + 10w + 200$$

$$= 1800 + \frac{36000}{w} + 10w + 200$$

$$= 2000 + \frac{36000}{w} + 10w$$

$$A' = -36000w^{-2} + 10$$

$$-\frac{36000}{w^2} + 10 = 0$$

$$-36000 = -10w^2$$

$$w^2 = 3600$$

$$w = \cancel{\pm} 60 \qquad w = 60 \text{ ft}$$

(-60 doesn't make sense)

$$w = 60, \quad l = \frac{1800}{60} = 30$$

So the smallest dimensions are

$$60+20 \times 30+10$$

$$\boxed{80 \text{ ft} \times 40 \text{ ft}}$$