Math 122: Exam 1 Practice Problems

1. (a) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m. $\quad y - y_1 = m(x - x_1)$

   (b) State the Slope-Intercept form of a line with slope m and y-intercept b. $\quad y = mx + b$

2. Let f be a function and let $a < b$ be given. State the average rate of change of f on the interval $[a, b]$. $\quad \dfrac{f(b) - f(a)}{b - a}$

3. Given a quantity P, state the relative change of the quantity from P to P'. $\quad \dfrac{P' - P}{P} \left(\dfrac{\text{New} - \text{Old}}{\text{Old}} \right)$

4. (a) State the form of an exponential function of a variable t with initial value P_0 and base a:

   $$P(t) = \underline{\quad P_0\, a^t \quad\quad\quad}.$$

   (b) The relative rate of change of P is

   $$r = \underline{\quad a - 1 \quad\quad\quad}.$$

   [Hint: If you don't recall the formula, this is just the relative change from $P(t)$ to $P(t+1)$.]

   (c) The function P models

   (i) exponential growth when r is $\underline{\quad \text{positive} \quad\quad}$.

   (ii) exponential decay when r is $\underline{\quad \text{negative} \quad\quad}$.

   (d) The continuous growth/decay rate is

   $$k = \underline{\quad \ln(a) \quad\quad\quad}.$$

5. Let $0 < x$, $0 < y$ be given. Fill in the blanks:

   (i) $\ln(1) = \underline{\quad 0 \quad\quad}$

   (iv) $\ln(x^r) = \underline{\quad r\ln(x) \quad\quad}$

   (ii) $\ln(xy) = \underline{\quad \ln(x) + \ln(y) \quad}$

   (v) $\ln(e^x) = \underline{\quad x \quad\quad}$

   (iii) $\ln\left(\dfrac{x}{y}\right) = \underline{\quad \ln(x) - \ln(y) \quad}$

   (vi) $e^{\ln(x)} = \underline{\quad x \quad\quad}$

6. (a) Find the slope of the line passing through the points $(3, \frac{1}{2})$ and $(2, 1)$. $m = -\frac{1}{2}$

   (b) Write the equation of this line in Point-Slope Form. $y - 1 = -\frac{1}{2}(x - 2)$

   (c) Write the equation of this line in Slope-Intercept Form. $y = -\frac{1}{2}x + 2$

   (d) Sketch a graph of $f(x)$. Label the x-intercept and the y-intercept.
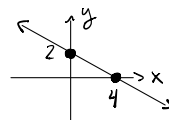
7. Let $f(x) = -x^2 + 1$.

   (a) Compute the average rate of change for f between $x = 3$ and $x = 5$. -8

   (b) Give the Point-Slope form of the line that passes through $(3, f(3))$ and $(5, f(5))$. $y + 8 = -8(x - 3)$

   (c) Give the Slope-Intercept form of the line from part (b). $y = -8x + 16$

8. A biologist observes a population with initial size 9. In two years, the biologist returns to observe the population again and finds that there are 81.

   (a) Find an exponential function for the size of the population as a function of t years since the initial observation. $P(t) = 9(3)^t$

   (b) Does the function from part (a) model growth or decay? Growth

   (c) Use the model from part (a) to determine how many years it will take for the size of the population to reach 243. $t = 3$

9. A bank is offering an account that pays 7% interest compounded continuously. If you decide to invest money in this account, how long will it take for your initial investment to double? Round to the nearest year. $2 = e^{.07t}$, $t \approx 10$ yrs

10. A company hosts a weekly event. They find that 25 people attend at a ticket price of \$30, and 15 people attend at a ticket price of \$50. Assuming this relationship is linear, determine the ticket price that will generate the highest revenue. State the maximum revenue.

    $(30, 25)$ $(50, 15)$
    $m = -\frac{1}{2}$ $q - 15 = -\frac{1}{2}(p - 50)$
    $q = -\frac{1}{2}p + 40$
    $R(p) = p \cdot q = p(-\frac{1}{2}p + 40) = -\frac{1}{2}p^2 + 40p$ Maximum Revenue is \$800 with a price of \$40.

    Hint: First, find the equation of the line representing the quantity of tickets sold, q, in terms of price, p.

11. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars rent for \$50 a day and 10 cents a mile.

    (a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled. $C = 40 + 0.15m$ $C = 50 + .10m$

    (b) How should you decide which company is cheaper? It depends on number of miles driven. $m < 200$, Company A $m > 200$, Company B

12. Which relative change is larger: An increase in class size from 5 to 10 or 30 to 50?

    5 to 10 is a 100% increase
    while 30 to 50 is a 66.7% increase

13. Let $f(t)$ be the number of US billionaires in year t. Express the following statements in terms of f.

    (a) In 1985 there were 13 US billionaires. $f(1985) = 13$

    (b) In 1990 there were 99 US billionaires. $f(1990) = 99$

    (c) Find the average yearly increase in the number of US billionaires between 1985 and 1990. Express this using f. 17.2

    (d) Assuming the yearly increase remains constant, find a formula predicting the number of US billionaires in year t. $f(t) = 17.2t - 34,129$

14. The demand curve of a product is given by $q = 120,000 - 500p$ and the supply curve is given by $q = 1000p$, where price is in dollars.

    (a) At a price of \$100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?   Consumers will buy 70,000 units; Producers will supply 100,000 units; Down

    (b) Find the equilibrium price and quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to equilibrium price? $\$80$, yes

15. Let $f(x) = x^2$, $g(x) = 1/x$, $h(x) = \sqrt{x-4}$ and $\ell(x) = 3x + 2$.

    (a) Find $f \circ \ell(x)$. $(3x+2)^2$

    (b) Find $g(f(x))$. $1/x^2$

    (c) Find $h(g(1/8))$. 2

    (d) Find $\ell(g(3))$. 3

    (e) Find $g \circ h(x)$. $\dfrac{1}{\sqrt{x-4}}$

    (f) Find $\ell(h(x+4))$. $3\sqrt{x} + 2$

16. Use the variable u for the inside function to express each of the following as a composite function.

    (a) $C = 12 \ln (q^3 + 1)$ $u = q^3 + 1$, $C = 12 \ln(u)$

    (b) $P = 16e^{-0.6t}$ $u = -0.6t$, $P = 16 e^u$

    (c) $y = (5t^2 - 2)^6$ $u = 5t^2 - 2$, $P = u^6$