Sols

PRODUCT RULE AND QUOTIENT RULE

Differentiate. Use proper notation and simplify your final answers. In some cases it might be advantageous to simplify/rewrite first. Do not use rules found in later sections.

1. $f(x)=(1+\sqrt{x})(x^3)$

$f(x)=x^3+x^{7/2}$

$f'(x)=3x^2+\frac{7}{2}x^{5/2}$

2. $g(t)=\left(\frac{2}{t}+t^5\right)(t^3+1)$

$g(t)=(2t^{-1}+t^5)(t^3+1)$

$g'(t)=(-2t^{-2}+5t^4)(t^3+1)+3t^2(2t^{-1}+t^5)$

3. $h(y)=\dfrac{1}{y^3+2y+1}$

$$h'(y)=\frac{0-1(3y^2+2)}{(y^3+2y+1)^2}$$

$$=\frac{-(3y^2+2)}{(y^3+2y+1)^2}$$

4. $y=\dfrac{1}{x+\sqrt{x}}$

$$y'=\frac{0-1(1+\frac{1}{2}x^{-1/2})}{(x+x^{1/2})^2}$$

$$=\frac{-(1+\frac{1}{2}x^{-1/2})}{(x+x^{1/2})^2}$$

5. $y=2^x e^x$

$y'=\ln(2)2^x e^x+2^x e^x$

6. $g(z)=\dfrac{z^2+1}{z^3-5}$

$$g'(z)=\frac{2z(z^3-5)-(z^2+1)(3z^2)}{(z^3-5)^2}$$

$$=\frac{2z^4-10z-3z^4-3z^2}{(z^3-5)^2}$$

$$=\frac{-z^4-3z^2-10z}{(z^3-5)^2}$$

7. $y=\dfrac{\sqrt{x}}{x^3+1}$

$$y'=\frac{\frac{1}{2}x^{-1/2}(x^3+1)-x^{1/2}(3x^2)}{(x^3+1)^2}$$

$$=\frac{\frac{1}{2}x^{-1/2}(x^3+1)-3x^{5/2}}{(x^3+1)^2}$$

8. $z=\dfrac{t^2}{(t-4)(2-t^3)}=\dfrac{t^2}{2t-t^4-8+4t^3}$

$$z'=\frac{2t(2t-t^4-8+4t^3)-t^2(2-4t^3+12t^2)}{(2t-t^4-8+4t^3)^2}$$

$$=\frac{4t^2-2t^5-16t+8t^4-2t^2+4t^5-12t^4}{(2t-t^4-8+4t^3)^2}$$

$$=\frac{2t^5-4t^4+2t^2-16t}{(2t-t^4-8+4t^3)^2}$$

9. $h(x)=\frac{(x^3+1)\sqrt{x}}{x^2} = \frac{x^{7/2}+x^{1/2}}{x^2} = x^{3/2}+x^{-3/2}$

$$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-5/2}$$

10. $y(m)=\frac{(e^m)(\sqrt[3]{m})}{m^2+3}$

$$y'(m) = \frac{\left[e^m m^{1/3} + \frac{1}{3}m^{-2/3}e^m\right](m^2+3) - e^m m^{1/3}(2m)}{(m^2+3)^2}$$

$$= \frac{\left(e^m m^{1/3} + \frac{1}{3}m^{-2/3}e^m\right)(m^2+3) - 2e^m m^{4/3}}{(m^2+3)^2}$$

11. $g(x)=(x+\sqrt{x})(3^x)$

$$g'(x) = (1+\tfrac{1}{2}x^{-1/2})(3^x) + (x+x^{1/2})\ln(3)3^x$$

12. Let $f(x)=g(x)h(x)$, $g(10)=-4$, $h(10)=560$, $g'(10)=0$, and $h'(10)=35$. Find $f'(10)$.

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

$$f'(10) = g'(10)h(10) + g(10)h'(10) = 0(560) + -4(35) = -140$$

13. Let $y(x)=\frac{z(x)}{1+x^2}$, $z(-3)=6$, and $z'(-3)=15$. Find $y'(-3)$.

$$y'(x) = \frac{z'(x)(1+x^2) - z(x)(2x)}{(1+x^2)^2}$$

$$y'(-3) = \frac{z'(-3)(1+(-3)^2) - z(-3)(2(-3))}{(1+(-3)^2)^2}$$

$$= \frac{15(10)-(6)(-6)}{(10)^2} = \frac{150+36}{100} = \frac{186}{100} = 1.86$$