Practice Exam 2 Solutions

(1) $f(x) = 3x^5 - 5x^3$ on $-3 \le x \le 6$

$f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1)$

critical pts at $x = 0, 1, -1$

$f''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$

$f''(0) = 0$ neither
$f''(1) > 0$ min
$f''(-1) < 0$ max

inflection pts at $x = 0, \sqrt{1/2}, -\sqrt{1/2}$

$\boxed{f(-3) = -594}$ Global Min

$f(-1) = 2$ $\quad f(1) = -2$

$\boxed{f(6) = 22{,}248}$ Global Max

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(2) $g(t) = te^{-t}$ for $t > 0$

$g'(t) = e^{-t} + -te^{-t} = e^{-t}(1-t)$ Crit. pts at $t = 1$

$g''(t) = -e^{-t}(1-t) - e^{-t} = -e^{-t}(2-t)$ infl. pt at $t = 2$

$g''(1) < 0$ so local max

Global Min at $t = 0$

Global Max at $t = 1$

(3) $f(x) = x + \frac{1}{x}$ for $x > 0$

$f'(x) = 1 - \frac{1}{x^2}$ crit. pt. at $x = 1$

$f''(x) = \frac{2}{x^3}$ no infl. pts.

$f''(1) > 0$ local min

Since concave up everywhere on $x > 0$
1 is a global min.
There is no global max.

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(4) $h(w) = w - \ln w$ for $w > 0$

$h'(w) = 1 - \frac{1}{w}$ crit. pt. at $w = 1$.

$h''(w) = \frac{1}{w^2}$ no infl. pts.

$h''(1) = 1 > 0$ so local min.

Since concave up for $w > 0$, 1 is a global min.
There is no global max.

(5) $f(x)=x^3-3x^2-9x+15$ on $-5 \le x \le 4$.

$f'(x)=3x^2-6x-9=3(x^2-2x-3)=3(x-3)(x+1)$

crit. pts at $x=3,-1$.

$f''(x)=6x-6=6(x-1)$ infl. pt at $x=1$

$f''(3)>0$ local min

$f''(-1)<0$ local max

$\boxed{f(-5)=-140 \text{ Global Min}}$; $\boxed{f(-1)=20 \text{ Global Max}}$; $f(3)=-12$; $f(4)=-5$

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(6) $g(x)=x^3-3x^2$ on $-1 \le x \le 3$

$g'(x)=3x^2-6x=3x(x-2)$ crit. pts at $x=0,2$

$g''(x)=6x-6=6(x-1)$ infl. pt at $x=1$

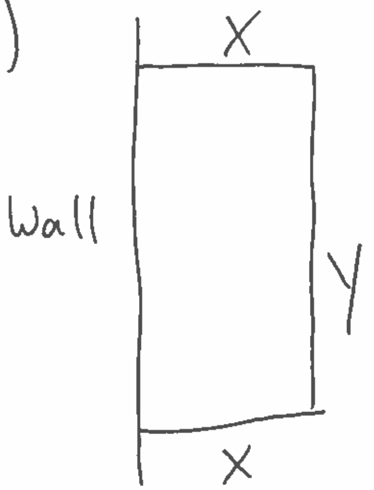
$g''(0)<0$ local max $g''(2)>0$ local min

$g(-1)=-4$; $g(0)=0$; $g(2)=-4$; $g(3)=0$

-1 and 2 are global mins

0 and 3 are global max.

(7)



$$100 = 2x + y \Rightarrow y = 100 - 2x$$

$$\text{Area} = A = x \cdot y$$

$$= x(100-2x)$$

$$= 100x - 2x^2.$$

$$\frac{dA}{dx} = 100 - 4x \quad \text{crit pt at } x = 25$$

$$\frac{d^2A}{dx^2} = -4 \quad \text{so} \quad x = 25 \text{ is a local max.}$$

When $x=25$, $y=50$. So max Area $= 25 \cdot 50 = 1{,}250$ sq. ft.

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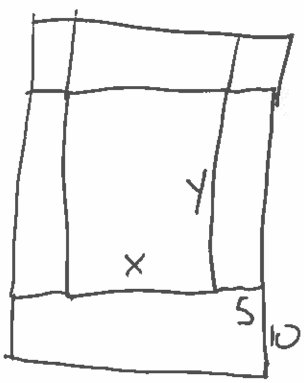
(8)



Area $= xy = 1800$

Total area (with decks) $= (x+5)(y+10)$

$= xy + 5y + 10x + 50$

$= 1850 + 5y + 10x.$

There is a mistake here. Where is it?

Want to minimize total area.

Since $xy = 1800 \Rightarrow y = \frac{1800}{x}$. So that

$$\text{Total Area} = TA = 1850 + \frac{9000}{x} + 10x.$$

$$\frac{dTA}{dx} = 10 - \frac{9000}{x^2} = 0$$ when $x = \pm 30$, but -30 does not make sense.

If $x=30$ then $y=60$. So property must be

70 by 35

(9) $q(t) = 20(e^{-t} - e^{-2t})$

$$q'(t) = 20(-e^{-t} + 2e^{-2t}) = 20e^{-t}(2e^{-t} - 1)$$

crit pt where $2e^{-t} - 1 = 0 \Rightarrow 2e^{-t} = 1 \Rightarrow e^{-t} = \frac{1}{2} \Rightarrow t = -\ln(\frac{1}{2})$
$= \ln(2)$

Max occurs at $t = \ln(2)$ with a value of 5 mg.

In the long run, your body will ~~approach~~ approach 0 mg.

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(10) Profit Maximized when $MP = MR - MC = 0$ or in other words when $MR = MC$.

$$0.03q^2 - 1.4q + 34 = 30$$

$$0.03q^2 - 1.4q + 4 = 0$$

$$\frac{1.4 \pm \sqrt{1.4^2 - 4(0.03)(4)}}{2(0.03)} = \frac{1.4 \pm 1.2165}{2(0.03)} = \quad 3.057 \text{ or } 44.36$$

(11) $f(2)=5$ and $f'(2)=0$.

$f(2)=a(2-b\ln 2)=5$ and $f'(x)=a(1-\frac{b}{x})$

So $f'(2)=a(1-\frac{b}{2})=0$.

Therefore $a=0$ or $b=2$. a cannot equal 0 or $f(2)\neq 5$.

So $b=2$. Thus

$f(2)=a(2-2\ln 2)=5$ and $a=\dfrac{5}{2-2\ln 2}$.

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(12) $f(x)=x^2+ax+b$

$f'(x)=2x+a$ $\quad$ $f'(3)=0$ so $6+a=0$ and $a=-6$.

$f(3)=5=9-6(3)+b \Rightarrow b=14$.

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(13) $f(x)=xe^{ax}$

$f'(x)=e^{ax}+axe^{ax}=e^{ax}(1+ax)$

$f'(3)=0=e^{3a}(1+3a) \Rightarrow a=-\frac{1}{3}$.

(13) For what value of a does $f(x) = xe^{ax}$ have a critical point at $x = 3$?

If you want more practice finding maxima and minima go back to the differentiation worksheet and try to find the local/global maxima and minima for those functions.

Implications for the Graph

For the following graphs determine the sign of the first and second derivative.