Practice Exam 2
Chapters 3 and 4

This test will be over exactly three things:

1. Taking derivatives
2. Using applications of the derivative
3. Recognizing graphically the implications of the first and second derivative

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Taking Derivatives

We have three basic functions that we have discussed in this class: power functions, exponential functions and the natural log function. You should know the rules for taking the derivatives of these three functions.

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(a^x) = \ln(a)a^x \qquad \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

Notice that above I have omitted the derivative of e^x because this is just a special example of an exponential function, i.e. $\ln(e) = 1$.
Once you have these rules down you can move on to the remaining three rules and you will be able to take the derivative of any reasonable function given to you with minimal resistance. These three rules are given below. For functions f and g we have

Chain Rule: $\frac{d}{dt}(f(g(t))) = f'(g(t)) \cdot g'(t)$

Product Rule: $(fg)' = f'g + fg'$

Quotient Rule: $(\frac{f}{g})' = \frac{f'g - fg'}{g^2}$

For practice with these rules simply print out a new copy of the differentiation worksheet given at the end of Chapter 3 and take all the derivatives asked. There are nearly thirty functions on that worksheet. If you finish that and are still uncomfortable you only need to open whatever copy of the book you have to a random page and take the derivative of the first function you see.

Applications of the Derivative

The only applications we have discussed in Chapter 4 involve finding critical points and inflection points and using these to find local (or global) maxima and minima. Section 4.4 discusses these application specifically in the setting of Profit, Cost and Revenue functions.

Exercises: For the following exercises, find all critical points and inflection points and determine which of the critical points are local maxima or minima. Then determine the global maxima and minima on the indicated interval.

(1) $f(x) = 3x^5 - 5x^3$ on $-3 \leq x \leq 6$

(2) $g(t) = te^{-t}$ for $t > 0$

(3) $f(x) = x + \frac{1}{x}$ for $x > 0$

(4) $h(w) = w - \ln w$ for $w > 0$

(5) $f(x) = x^3 - 3x^2 - 9x + 15$ on $-5 \leq x \leq 4$

(6) $g(x) = x^3 - 3x^2$ on $-1 \leq x \leq 3$

Word Problems:

(7) You have 100 feet of fencing and want to enclose a rectangular area up against a long, straight wall, what is the largest area you can enclose?

(8) A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot-wide decks along either side and 10-foot-wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.

(9) The quantity of a drug in the bloodstream t hours after a tablet is swallowed is given, in mg, by $q(t) = 20(e^{-t} - e^{-2t})$. When is the maximum quantity of drug in the bloodstream and what is the maximum? In the long run, what happens to the quantity?

(10) The marginal cost and marginal revenue of a company are $MC(q) = 0.03q^2 - 1.4q + 34$ and $MR(q) = 30$, where q is the number of items manufactured. For what quantities is profit maximized?

Understanding Concepts:

(11) For what values of a and b does $f(x) = a(x - b\ln x)$ have a local minimum at $(2, 5)$?

(12) For what values of a and b does $f(x) = x^2 + ax + b$ have a local maximum at $(3, 5)$?

(13) For what value of a does $f(x) = xe^{ax}$ have a critical point at $x = 3$?

If you want more practice finding maxima and minima go back to the differentiation worksheet and try to find the local/global maxima and minima for those functions.

Implications for the Graph

For the following graphs determine the sign of the first and second derivative.