MATH 122

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Calculus for Business Administration and Social Sciences

Outline

1. 6.3: Using the Fundamental Theorem to Compute Definite Integrals

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Example

Compute

$$\int_1^3 2x \, dx.$$

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$$\int_1^3 2x\,dx.$$

$$\int_1^3 2x\,dx \quad =$$

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$$\int_1^3 2x\,dx.$$

$$\int_1^3 2x\,dx = 2\int_1^3 x\,dx$$

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$$\begin{aligned}\int_1^3 2x\,dx &= 2\int_1^3 x\,dx \\ &= 2\left[\frac{1}{2}x^2\right]_1^3\end{aligned}$$

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$$\begin{aligned}\int_1^3 2x\,dx &= 2\int_1^3 x\,dx \\ &= 2\left[\frac{1}{2}x^2\right]_1^3 \\ &= x^2\Big|_1^3 \\ &= 3^2 - 1^2\end{aligned}$$

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\int_1^3 2x\,\mathrm{d}x &= 2\int_1^3 x\,\mathrm{d}x \\
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&= x^2\Big|_1^3 \\
&= 3^2 - 1^2 \\
&= 9 - 1 \\
&= 8.
\end{aligned}$$

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$$\int_0^2 6x^2 \, \mathrm{d}x = 6 \int_0^2 x^2 \, \mathrm{d}x$$

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$$\begin{aligned}\int_0^2 6x^2\,\mathrm{d}x &= 6\int_0^2 x^2\,\mathrm{d}x \\ &= \frac{6}{3}x^3\Big|_0^2\end{aligned}$$

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$$\begin{aligned}\int_0^2 6x^2\,\mathrm{d}x &= 6\int_0^2 x^2\,\mathrm{d}x \\ &= \frac{6}{3}x^3\Big|_0^2 \\ &= 2\left(2^3 - 0^3\right)\end{aligned}$$

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$$\begin{aligned}\int_0^2 t^3\,\mathrm{d}t &= \frac{1}{4}t^4\Big|_0^2 \\ &= \frac{1}{4}(16-0) \\ &= 4.\end{aligned}$$

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$$\begin{aligned}\int_1^2 8x + 5\,dx &= 8\int_1^2 x\,dx + 5\int_1^2 dx \\ &= \frac{8}{2}x^2\Big|_1^2 + 5x\Big|_1^2\end{aligned}$$

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$$\begin{aligned}
\int_1^2 8x + 5\,dx &= 8\int_1^2 x\,dx + 5\int_1^2 dx \\
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&= 4(4-1) + 5(2-1)
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The method of Integration by Substitution is intended to integrate a product of functions that appears to have come from an application of the chain rule.

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Let $u = x^2 + 4$ so

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$$\begin{aligned}\int \frac{2x}{x^2+4}\,dx &= \int \frac{1}{u}\,du \\ &= \ln|u| + c \\ &= \ln\left|x^2+4\right| + c \\ &= \ln(x^2+4) + c.\end{aligned}$$

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Let $u = t^2 + 1$ so $du = 2t\,dt$.

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Let $u = t^2 + 1$ so $\mathrm{d}u = 2t\,\mathrm{d}t$. Since we want to replace $t\,\mathrm{d}t$, we divide both sides by 2 to get $\mathrm{d}u/2 = t\,\mathrm{d}t$.

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$$\int te^{t^2+1}\,\mathrm{d}t = \int e^u \frac{\mathrm{d}u}{2}$$

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$$\int te^{t^2+1}\,dt.$$

Let $u = t^2 + 1$ so $du = 2t\,dt$. Since we want to replace $t\,dt$, we divide both sides by 2 to get $du/2 = t\,dt$. Therefore

$$\begin{aligned}\int te^{t^2+1}\,dt &= \int e^u \frac{du}{2} \\ &= \frac{1}{2}\int e^u\,du\end{aligned}$$

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$$\begin{aligned}\int te^{t^2+1}\,\mathrm{d}t &= \int e^u \frac{\mathrm{d}u}{2} \\ &= \frac{1}{2}\int e^u\,\mathrm{d}u \\ &= \frac{1}{2}e^u + c\end{aligned}$$

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Let $u = t^2 + 1$ so $\mathrm{d}u = 2t\,\mathrm{d}t$. Since we want to replace $t\,\mathrm{d}t$, we divide both sides by 2 to get $\mathrm{d}u/2 = t\,\mathrm{d}t$. Therefore

$$\begin{aligned}
\int te^{t^2+1}\,\mathrm{d}t &= \int e^u \frac{\mathrm{d}u}{2} \\
&= \frac{1}{2}\int e^u\,\mathrm{d}u \\
&= \frac{1}{2}e^u + c \\
&= \frac{1}{2}e^{t^2+1} + c.
\end{aligned}$$

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$$\int x^3 \sqrt{x^4 + 5}\, dx.$$

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$$\int x^3\sqrt{x^4+5}\,\mathrm{d}x \quad =$$

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$$\int x^3 \sqrt{x^4 + 5}\,\mathrm{d}x.$$

Let $u = x^4 + 5$ so $\mathrm{d}u = 4^3 x\,\mathrm{d}x$. Hence $\mathrm{d}u/4 = x^3\,\mathrm{d}x$. Therefore

$$\int x^3 \sqrt{x^4 + 5}\,\mathrm{d}x = \int \sqrt{u}\frac{\mathrm{d}u}{4}$$

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Compute

$$\int x^3 \sqrt{x^4 + 5}\, dx.$$

Let $u = x^4 + 5$ so $du = 4^3 x\, dx$. Hence $du/4 = x^3\, dx$. Therefore

$$\begin{aligned} \int x^3 \sqrt{x^4 + 5}\, dx &= \int \sqrt{u} \frac{du}{4} \\ &= \frac{1}{4} \int u^{\frac{1}{2}}\, du \end{aligned}$$

Example

Compute
$$\int x^3\sqrt{x^4+5}\,\mathrm{d}x.$$

Let $u = x^4 + 5$ so $\mathrm{d}u = 4^3 x\,\mathrm{d}x$. Hence $\mathrm{d}u/4 = x^3\,\mathrm{d}x$. Therefore

$$\begin{aligned}\int x^3\sqrt{x^4+5}\,\mathrm{d}x &= \int \sqrt{u}\frac{\mathrm{d}u}{4}\\ &= \frac{1}{4}\int u^{\frac{1}{2}}\,\mathrm{d}u\\ &= \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}\right)+c\end{aligned}$$

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Example

Compute

$$\int \frac{t^2}{1+t^3}\,\mathrm{d}t.$$

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$$\int \frac{t^2}{1+t^3}\,\mathrm{d}t = \int \frac{1}{u}\frac{\mathrm{d}u}{3}$$

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$$\begin{aligned}
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&= \frac{1}{3}\int \frac{\mathrm{d}u}{u} \\
&= \ln|u| + c \\
&= \ln\left|1 + t^3\right| + c.
\end{aligned}$$