Math 122

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Calculus for Business Administration and Social Sciences

Outline

Definition 1

Given a function, f, we say an *antiderivative* of f is a function F such that

$$F'(x) = f(x).$$

Remarks

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1. By the Fundamental Theorem of Calculus, given an antiderivative, F, of f,

$$\int_a^b f(x)\,\mathrm{d}x =$$

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2. If f admits an antiderivative, F, then for any $c \in \mathbb{R}$, $F(x) + c$ is also an antiderivative because

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$$\frac{\mathrm{d}}{\mathrm{d}x}(F(x)+c) = \frac{\mathrm{d}}{\mathrm{d}x}F(x) + \frac{\mathrm{d}}{\mathrm{d}x}(c) = f(x) + 0 = f(x).$$

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Polynomials

Since we know the antiderivative for a monomial, given a polynomial

$$f(x) = \sum_{i=0}^{n} a_{n-i} x^{n-i}$$

we have the antiderivative

$$F(x) = \sum_{i=0}^{n} a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c,\ c \in \mathbb{R}.$$

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Let $P(x) = P_0 e^{kx}$. Since $\frac{\mathrm{d}}{\mathrm{d}x} e^{kx} = ke^{kx}$, we observe that

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This implies

$$\frac{P_0}{k} e^{kx} + c, \; c \in \mathbb{R}$$

is an antiderivative of $P(x)$.

Exponents with Arbitrary Base

If we want to integrate $P(x) = P_0 a^x$, then we can rewrite as

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Therefore

$$\frac{P_0}{\ln(a)} a^x + c, \; c \in \mathbb{R}$$

is an antiderivative of $P(x)$.

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If f is a function with an antiderivative F, then the *indefinite integral* is the family of functions

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Remark 2

Note that this immediately implies

$$\frac{\mathrm{d}}{\mathrm{d}x}\int f(x)\,\mathrm{d}x = f(x).$$

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Properties of the Indefinite Integral

Assume that $\int f(x)\,dx$ and $\int g(x)\,dx$ exist. Then

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2. Constants pass through the indefnite integral:

$$\int af(x)\,\mathrm{d}x = a\int f(x)\,\mathrm{d}x,\ a \in \mathbb{R}.$$

Examples

Integrate

(A) x^5 (B) t^8 (C) $12x^3$ (D) $q^3 - 6q^2$

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$$\int t^8 \, \mathrm{d}t = \frac{1}{9}x^9 + c.$$

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$$\int 12x^3 \, dx \quad =$$

EXAMPLES

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$$\begin{aligned}\int 12x^3\,dx &= 12\int x^3\,dx \\ &= 12\frac{1}{4}x^4 + c\end{aligned}$$

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&= \int q^3 \, dq - 6 \int q^2 \, dq \\
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&= \frac{1}{4}q^4 - 2q^3 + c.
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Example

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$$\begin{aligned}\int 12e^{0.2t}\,\mathrm{d}t &= 12\int e^{\frac{t}{5}}\,\mathrm{d}t \\ &= 12\left(5e^{\frac{t}{5}}\right) + c \\ &= 60e^{.02t} + c.\end{aligned}$$

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Hence

$$\int \frac{dx}{x^n} = \frac{1}{-n+1}x^{-n+1} + c = \frac{1}{(-n+1)x^{n-1}} + c$$

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$$\begin{aligned} \int \frac{dx}{x^3} &= \int x^{-3}\,dx \\ &= \frac{1}{-2}x^{-2} + c \\ &= \frac{-1}{2x^2} + c. \end{aligned}$$

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so we would expect

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- This isn't quite true.

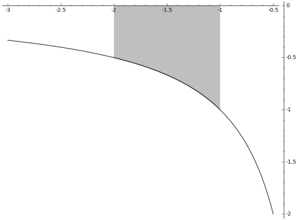
The Indefinite Integral of $\frac{1}{x}$ (Cont.)

Let $F(x)$ be an antiderivative of $1/x$.

The Indefinite Integral of $\frac{1}{x}$ (Cont.)

Let $F(x)$ be an antiderivative of $1/x$. Since this function is continuous away from $x = 0$, we could ask:

> What is the area between $1/x$ and the x-axis from $x = -2$ to $x = -1$?



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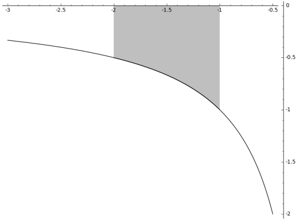

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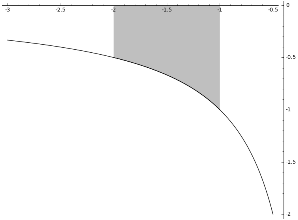

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$$\int_{-2}^{-1} \frac{dx}{x} = F(-1) - F(-2).$$

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The Indefinite Integral of $\frac{1}{x}$ (Cont.)

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Therefore

$$\int \frac{dx}{x} = \ln|x| + c.$$