MATH 122

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Calculus for Business Administration and Social Sciences

Outline

1. 3.1: Derivatives of Polynomials
   - Constants
   - Linearity
   - Power Rule

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Derivative of Constants

- Let $f(x) = a$ for $a \in \mathbb{R}$ (this means “a is an element of $\mathbb{R}$”, the set of real numbers).

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- Therefore $f'(x) = 0$.

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- $$\frac{d}{dx}(af(x))$$

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- $$\frac{\mathrm{d}}{\mathrm{d}x}\left(af(x)\right) = a\frac{\mathrm{d}}{\mathrm{d}x}f(x).$$

Derivative of a Power Functions

The derivative of x^n for $n \in \mathbb{R}$ is

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Remark 1

The derivative of a linear function is

$$\frac{d}{dx}(mx + b) = m\frac{d}{dx}x + \frac{d}{dx}(b) = m$$

Derivative of Polynomials

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The derivative is

$$\begin{aligned} p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0) \\ &= a_n \frac{\mathrm{d}}{\mathrm{d}x}(x^n) + a_{n-1} \frac{\mathrm{d}}{\mathrm{d}x}(x^{n-1}) + \cdots \\ &+ a_2 \frac{\mathrm{d}}{\mathrm{d}x}(x^2) + a_1 \frac{\mathrm{d}}{\mathrm{d}x}(x) \end{aligned}$$

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$$\begin{aligned} p'(x) &= \frac{d}{dx}(a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0) \\ &= a_n \frac{d}{dx}(x^n) + a_{n-1} \frac{d}{dx}(x^{n-1}) + \cdots \\ &+ a_2 \frac{d}{dx}(x^2) + a_1 \frac{d}{dx}(x) \\ &= n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots + 2 a_2 x + a_1. \end{aligned}$$

EXAMPLE

Differentiate the following

1. $A(t) = 3t^5$
2. $r(p) = p^5 + p^3$
3. $f(x) = 5x^2 - 7x^3$
4. $g(t) = \frac{t^2}{4} + 3$

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&= 15t^4.
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$$\begin{aligned} r'(p) &= \frac{d}{dp}\left(p^5 + p^3\right) \\ &= \frac{d}{dp}\left(p^5\right) + \frac{d}{dp}\left(p^3\right) \end{aligned}$$

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$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^3 - 2x^2 - 5x + 7) \\ &= \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(7) \end{aligned}$$

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Exponentials

The derivative of e^x is

$$\frac{\mathrm{d}}{\mathrm{d}x}(e^x) = e^x.$$

Logarithms

- The derivative of the natural logarithm is

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$$\begin{aligned}\frac{d}{dx}(\log_a(x)) &= \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right) \\ &= \frac{1}{\ln(a)}\frac{d}{dx}(\ln(x))\end{aligned}$$

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