MATH 122

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Calculus for Business Administration and Social Sciences

Outline

Definition

Definition 1

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- In this case, we can define a function $f'(x)$ that outputs the instantaneous rate of change of f at x.
- We call $f'(x)$ the *derivative function*.

THE TANGENT LINE

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- We call the line

$$y - f(x_0) = f'(x_0)(x - x_0)$$

the *line tangent to f at* $(x_0, f(x_0))$.

Linearization

- Since we defined $f'(x_0)$ by a limit,

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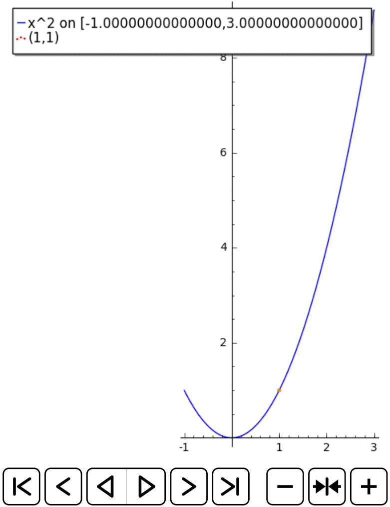
- This means f locally looks like a line!

Animation



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Consider the absolute value function

$$|x| = \begin{cases} x & \text{if } 0 \leq x, \\ -x & \text{else} \end{cases}$$

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- So the derivative at $(0, 0)$ is **not** defined: it's -1 if we approach from left to right, and 1 if right to left.

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- $0 \leq f'(x)$, then f is increasing on (a, b),
- $f'(x) = 0$, then f is constant on (a, b).