Math 122

Clifton

1.1: Functions
Graphs
1.2: Linear Functions
1.3: Average Rate of Change and Relative Change

Math 122

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Calculus for Business Administration and Social Sciences

1.1: Functions
Graphs

1.2: Linear Functions

1.3: Average Rate of Change and Relative Change

Outline

1 1.1: Functions
- Graphs

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1. 1.1: FUNCTIONS
   - Graphs
2. 1.2: LINEAR FUNCTIONS
3. 1.3: AVERAGE RATE OF CHANGE AND RELATIVE CHANGE

Definition

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1.1: Functions

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Definition 1

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Notation: A function named f that takes as input the *independent variable*, x, and outputs the *dependent variable*, y, is written as

$$y = f(x).$$

Example (Discrete Function)

Given any two sets we can define a function.

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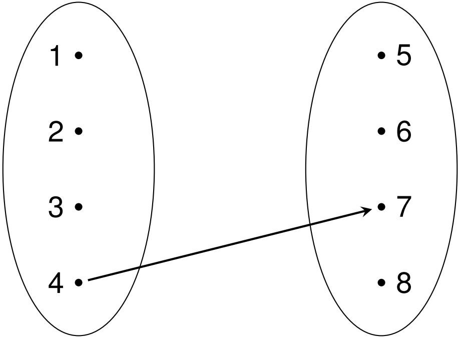

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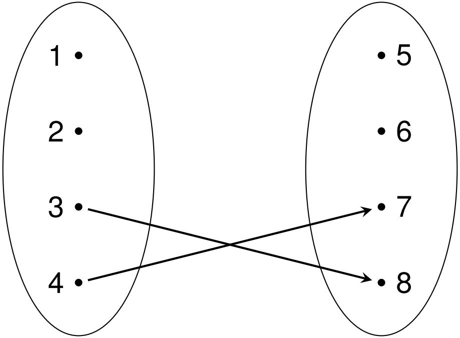

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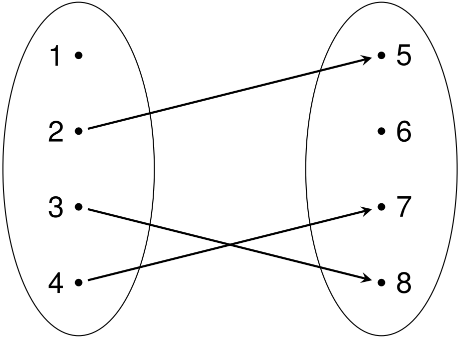

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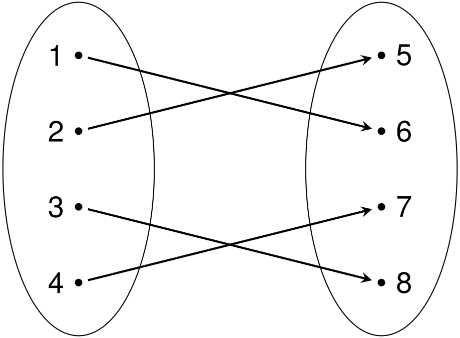

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- The domain of f is the set of all real numbers, $\mathbb{R}$.
- The range of f is the set of all non-negative real numbers,
$$\{x \in \mathbb{R} \mid 0 \leq x\}.$$

Non-Example

The following depicts a non-function.

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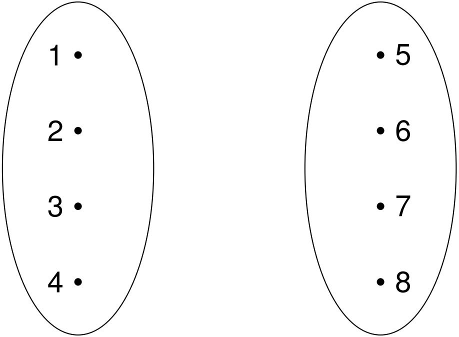

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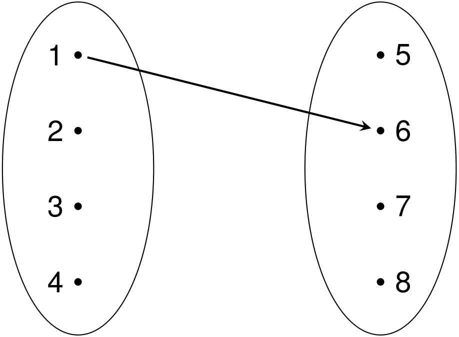

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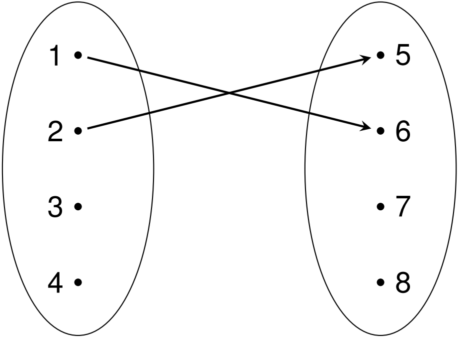

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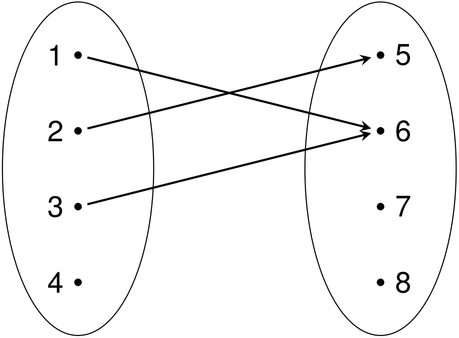

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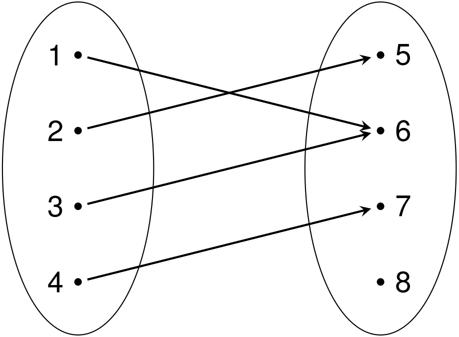

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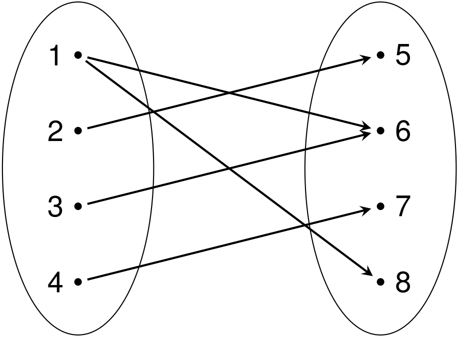

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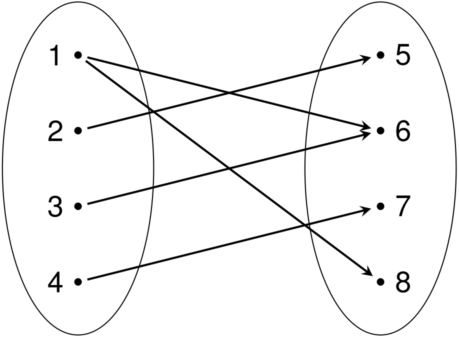

The value $f(1)$ is not well-defined because it requires a choice: it could be either 6 or 8.

Cartesian Plane

Recall that the *Cartesian plane* is the set of all pairs

$$\mathbb{R}^2 = \{(x, y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}.$$

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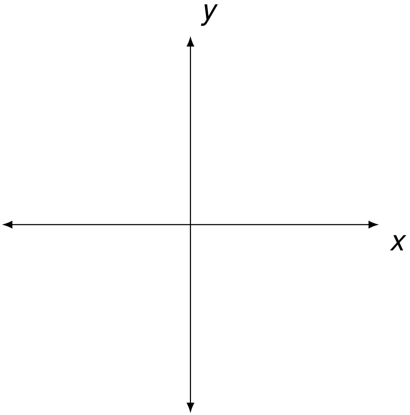
It can be depicted as



GRAPH OF A FUNCTION

DEFINITION 2

The graph of a real-valued function, f, with domain $D \subseteq \mathbb{R}$ is the set of pairs

$$\{(x, f(x)) \mid x \in D\} \subseteq \mathbb{R}^2.$$

It can be drawn on the Cartesian plane.

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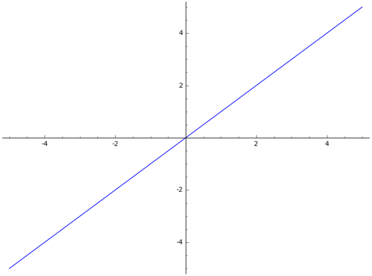

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We say that f is increasing/decreasing if it is increasing/decreasing on its entire domain.

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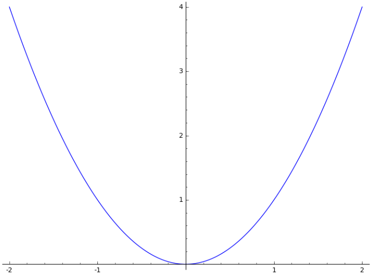

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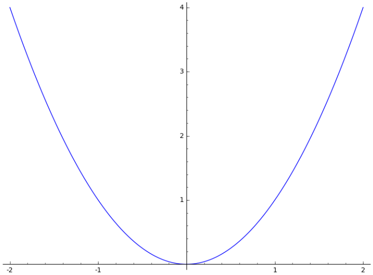

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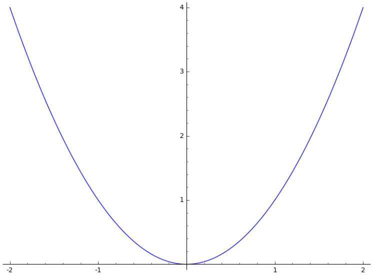

- Increasing on: $(0, \infty)$
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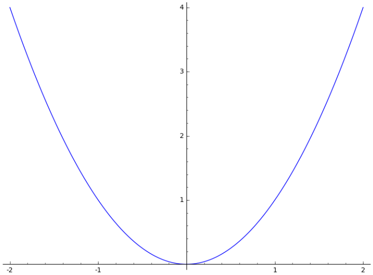

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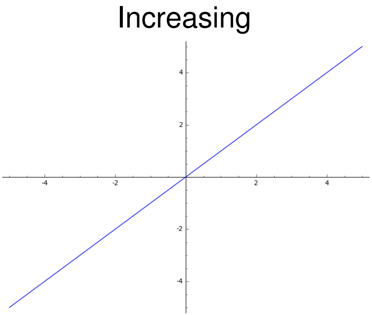

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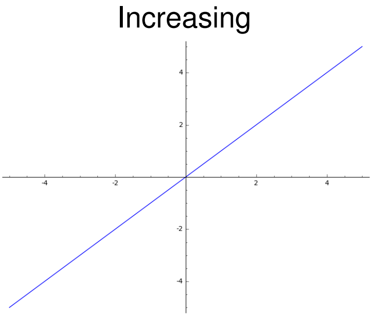

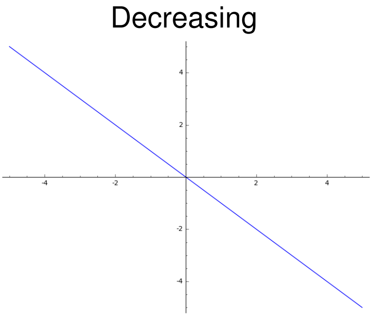

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Definition 4

Let f be a function of a real variable, x.

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Intercepts

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- The *y-intercept* is the point $(0, f(0))$ on the graph.

Example

Let $f(x) = x - 1$.

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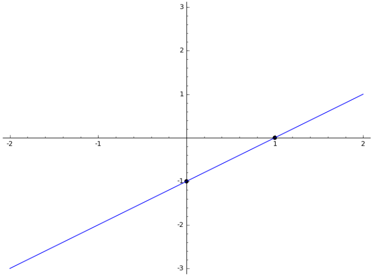

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- Linear functions have domain and range $\mathbb{R}$,
- The number m is called the *slope* of the f,
- The number b is the y-intercept,
- This form is usually called the *Slope-Intercept Form* of a line.

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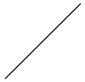

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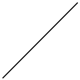

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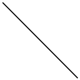

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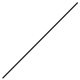

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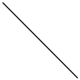

- Horizontal ($m = 0$):

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Point-Slope Form

Definition 6

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POINT-SLOPE FORM

DEFINITION 6

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Point-Slope Form

Definition 6

Given:

- a point, (x_0, y_0),
- a slope, m,

the equation of the line through (x_0, y_0) with slope m is

$$y - y_0 = m(x - x_0).$$

Given two points, (x_0, y_0) and (x_1, y_1), the slope of the line passing through them is

$$m = \frac{y_0 - y_1}{x_0 - x_1} = \frac{y_1 - y_0}{x_1 - x_0}.$$

Two Points Determine a Line

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To see these are the same line, put them both into Slope-Intercept Form.

Two Points Determine a Line (Cont.)

$$y = mx - \frac{y_0 - y_1}{x_0 - x_1}x_0 + y_0$$

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Two Points Determine a Line (Cont.)

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Difference Quotients

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Let f be a function.

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Let f be a function. Given x_0, x_1 in the domain of f, the *difference quotient* is

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This is just the slope of the line through $(x_0, f(x_0))$ and $(x_1, f(x_1))$.

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This is just the slope of the line through $(x_0, f(x_0))$ and $(x_1, f(x_1))$. This line is usually called the *Secant Line*.

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Difference Quotients for Linear Functions

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Difference Quotients for Linear Functions

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Let $f(x) = mx + b$. Given x_0 and x_1:

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{mx_1 + b - (mx_0 + b)}{x_1 - x_0}$$

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Graphs
1.2: Linear Functions
1.3: Average Rate of Change and Relative Change

Let $f(x) = mx + b$. Given x_0 and x_1:

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{mx_1 + b - (mx_0 + b)}{x_1 - x_0} = \frac{mx_1 - mx_0 + b - b}{x_1 - x_0}$$

Difference Quotients for Linear Functions

Let $f(x) = mx + b$. Given x_0 and x_1:

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Hence for a linear function, the difference quotient is just the slope.

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Let $f(x) = x^2$. For $x_0 = -1$, $x_1 = 2$:

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Difference Quotients for Non-Linear Functions

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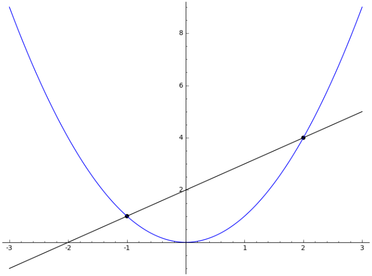
This is the slope of the secant line:



Difference Quotients for Non-Linear Functions (cont.)

For $x_0 = 0$, $x_1 = 2$:

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$$\frac{f(0) - f(2)}{0 - 2}$$

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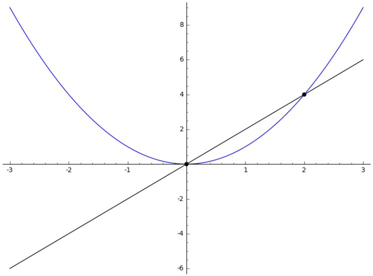

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DEFINITION 8

The *average rate of change* of a function f on an interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{f(a) - f(b)}{a - b}.$$

Average Rate of Change

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Remark 1

This is just the difference quotient from the last section.

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From Columbia, it's about 104 miles to Charleston.

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From Columbia, it's about 104 miles to Charleston. If you make the drive in two hours, what was your average speed? Take Columbia to be distance zero, and mark the starting time at $t = 0$. The average speed is:

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$$\frac{104 - 0}{2 - 0} = \frac{104}{2} = 52 \text{ mph.}$$

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From Columbia, it's about 104 miles to Charleston. If you make the drive in two hours, what was your average speed? Take Columbia to be distance zero, and mark the starting time at $t = 0$. The average speed is:

$$\frac{104 - 0}{2 - 0} = \frac{104}{2} = 52 \text{ mph}.$$

Remark 2

Note that this does not necessarily imply you drove 52 mph the entire time, but rather you averaged 52 mph.

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Find the average rate of change of $f(x) = \sqrt{x}$ on $[1, 4]$.

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Relative Change

Given a quantity, P, the *relative change* of the quantity from P to P' is

$$\frac{P' - P}{P}.$$

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If gas costs \$2.25 and the price increases by \$2, then find the relative change in price.

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If gas costs \$2.25 and the price increases by \$2, then find the relative change in price.

$$\frac{4.25 - 2.25}{2.25} = \frac{2}{2.25} = \frac{2}{\frac{9}{4}} = \frac{8}{9} = 0.\overline{8}.$$

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A pair of jeans costs 75.99 normally.

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EXAMPLE

A pair of jeans costs 75.99 normally. Today they are on sale for 52.99. What is the relative change in the price?

$$\frac{52.99 - 75.99}{75.99} = \frac{-23}{75.99} \approx -0.303.$$

Hence the price has been reduced by about 30%.

Example

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The number of sales per week for the jeans above is normally 25.

Example

The number of sales per week for the jeans above is normally 25. During the week the jeans are on sale, the number of weekly sales increases to 45.

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$$\frac{45 - 25}{25}$$

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$$\frac{45-25}{25} = \frac{20}{25}$$

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$$\frac{45-25}{25}=\frac{20}{25}=\frac{4}{5}.$$

Example

The number of sales per week for the jeans above is normally 25. During the week the jeans are on sale, the number of weekly sales increases to 45. Find the relative change in weekly sales.

$$\frac{45-25}{25} = \frac{20}{25} = \frac{4}{5}.$$

Hence weekly sales have increased by 80%.