Sols

Math 122: Exponential and Logarithmic Functions

1. Find the value in 8 years of a \$10,000 investment at a rate of 3% compounded continuously. $P(t)=P_0 e^{rt}$ $P(8)=10{,}000e^{.03(8)} = \$12{,}712.49$

2. You make an investment today at an interest rate of 4% compounded continuously in which you want to have \$8,000 in 5 years. How much do you need to invest today? $8000=P_0 e^{.04(5)}$ $P_0 = \frac{8000}{e^{.2}} = \$6{,}549.85$
   $P(t)=P_0 e^{rt}$

3. If \$12,000 is deposited in an account paying 8% interest per year, compounded annually, how long will it take for the balance to reach \$20,000?
   $P(t)=P_0(1+r)^t$ $20000 = 12000(1.08)^t$ $\frac{5}{3}=(1.08)^t$ $t=\frac{\ln(5/3)}{\ln(1.08)} \approx 6.64$ yrs

4. A person is to be paid \$2000 for work done over a year. Three payment options are to be considered. Option 1 is to pay the \$2000 in full now. Option 2 is to pay \$1000 now and \$1000 in a year. Option 3 is to pay the full \$2000 in a year. Assume an annual interest rate of 5% a year, compounded continuously. Without doing any calculations, which option is the best option financially for the worker? Explain your reasoning.
   It is best to receive \$2000 now so that in 1 year the entire amount gains interest.

5. Find the doubling time of a quantity that is increasing by 7% a year.
   $2=1(1.07)^t$ $t=\frac{\ln(2)}{\ln(1.07)} \approx 10.25$ years

6. A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

   (a) Write a formula for the amount, A mg, of caffeine in the body t hours after drinking a cup of coffee. $A(t)=100e^{-.17t}$

   (b) Find the half-life of caffeine. $50=100e^{-.17t}$ $\frac{1}{2}=e^{-.17t}$ $t=\frac{\ln(1/2)}{-.17} \approx 4.08$ hours

7. A firm decides to increase output at a constant relative rate from its current level of 20,000 to 30,000 units during the next five years. Calculate the annual percent rate of increase required to achieve this growth.
   $30000=20000a^5$ $\frac{3}{2}=a^5$ $a=\sqrt[5]{3/2}\approx 1.084$ $r=1.084-1=.084$ or 8.4%

8. During a recession a firm's revenue declines continuously so that the revenue, R (in millions), in t years' time is given by $R=5e^{-0.15t}$. After how many years will the revenue decline to 2.7 million?
   $2.7=5e^{-.15t}$ $\frac{2.7}{5}=e^{-.15t}$ $t=\frac{\ln(2.7/5)}{-.15}\approx 4.11$ years

9. The population of the US was 281.4 million in 2000 and 308.7 million in 2010. Assuming exponential growth,

   (a) In what year is the population expected to go over 350 million?

   (b) What population is predicted for the 2020 census?

(a) $308.7=281.4e^{10k}$

$\frac{308.7}{281.4}=e^{10k}$

$k=\frac{\ln\left(\frac{308.7}{281.4}\right)}{10}\approx 0.00926$

So, $350=281.4e^{0.00926t}$

$t=\frac{\ln(350/281.4)}{0.00926}\approx 23.56$ years

The population will reach 350 mil in 2023.

(b) $P(20)=281.4e^{0.00926(20}$

$= 338.7$ million