MATH 122: DEFINITE INTEGRAL DOMINOES - SOLUTIONS

ANN CLIFTON
UNIVERSITY OF SOUTH CAROLINA

Goal: To match definite integrals with their numerical or geometric representations.

(a) $\int_0^1 (x+1)(x-1)dx = \int_0^1 (x^2-1)dx = \frac{x^3}{3} - x|_0^1 = (\frac{1^3}{3} - 1) - (\frac{0^3}{3} - 0) = -\frac{2}{3}$

(b) $\int_0^1 (x - 8x^3)dx = \frac{x^2}{2} - 2x^4|_0^1 = (\frac{1^2}{2} - 2(1)^4) - (\frac{0^2}{2} - 2(0)^4) = -\frac{3}{2}$

(c) $\int_0^1 \frac{x^3-x}{x+1}dx = \int_0^1 \frac{x(x^2-1)}{x+1}dx = \int_0^1 \frac{x(x+1)(x-1)}{x+1}dx = \int_0^1 x(x-1)dx =$
$\int_0^1 (x^2 - x)dx = \frac{x^3}{3} - \frac{x^2}{2}|_0^1 = (\frac{1}{3} - \frac{1}{2}) - (0 - 0) = -\frac{1}{6}$

(d) $\int_0^1 e^{x+1}dx = e^{x+1}|_0^1 = e^2 - e^1 = e^2 - e$

(e) $\int_0^1 2x^3dx = \frac{x^4}{2}|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

(f) $\int_0^1 (x^2+1)dx = \frac{x^3}{3} + x|_0^1 = (\frac{1}{3} + 1) - (0 + 0) = \frac{4}{3}$

(g) $\int_0^1 x^2dx = \frac{x^3}{3}|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$

(h) $\int_0^1 \sqrt[3]{x}dx = \int_0^1 x^{1/3}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}|_0^1 = \frac{x^{4/3}}{4/3}|_0^1 = \frac{3}{4}x^{4/3}|_0^1 = \frac{3}{4}(1)^{4/3} - \frac{3}{4}(0)^{4/3} = \frac{3}{4}$

(i) $\int_0^1 \sqrt{x}dx = \int_0^1 x^{1/2}dx = \frac{x^{1/2+1}}{1/2+1}|_0^1 = \frac{2}{3}x^{3/2}|_0^1 = \frac{2}{3}(1)^{3/2} - 0 = \frac{2}{3}$

(j) $\int_0^1 \pi dx = \pi x|_0^1 = \pi(1) - \pi(0) = \pi$

(k) $\int_0^1 (x+1)^2 dx = \int_0^1 (x^2 + 2x + 1)dx = \frac{x^3}{3} + x^2 + x|_0^1 = (\frac{1^3}{3} + (1)^2 + 1) - (0) = \frac{7}{3}$

(l) $\int_0^1 (x+1)^3 dx = \int_0^1 (x+1)(x^2+2x+1)dx = \int_0^1 (x^3+3x^2+3x+1)dx = \frac{x^4}{4} + x^3 + \frac{3x^2}{2} + x|_0^1 = (\frac{1}{4} + 1 + \frac{3}{2} + 1) - (0) = \frac{15}{4}$