Sols

1. Let $f(x) = (3x^2 + 1)^2$. We are going to find the derivative of $f(x)$ in three ways and then compare the answers.

   (a) Algebraically multiply out the expression for $f(x)$ and then take the derivative.

   $$f(x) = (3x^2+1)(3x^2+1) = 9x^4+6x^2+1$$

   $$f'(x) = 36x^3+12x$$

   (b) View $f(x)$ as a product of two functions, $f(x) = (3x^2 + 1)(3x^2 + 1)$ and use the product rule to find $f'(x)$.

   $$\begin{aligned} f'(x) &= 6x(3x^2+1) + (3x^2+1)(6x) \\ &= 18x^3+6x+18x^3+6x \\ &= 36x^3+12x \end{aligned}$$

   (c) Apply the chain rule directly to the expression $f(x) = (3x^2 + 1)^2$

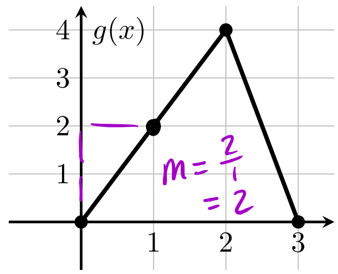
   $$\begin{aligned} f'(x) &= 2(3x^2+1)(6x) \\ &= 12x(3x^2+1) \\ &= 36x^3+12x \end{aligned}$$

   (d) Are your answers in parts a, b, and c the same? Why or why not?

   Yes! We should expect different methods to give equivalent results.

2. Let $f(x)$ and $g(x)$ be two functions. Values of $f(x)$ and $f'(x)$ are given in the table below and the graph of $g(x)$ is as shown.

x	1	2	3
$f(x)$	3	2	1
$f'(x)$	4	5	6



$g'(1) = 2$ Recall that the derivative at a point is the slope of the function at that point.

(a) Let $h(x) = g(f(x))$. Find $h'(3)$.

$h'(x) = g'(f(x))f'(x)$

$h'(3) = g'(f(3))f'(3) = g'(1)f'(3) = 2(6) = 12$

(b) Let $k(x) = f(g(x))$. Find $k'(1)$.

$k'(x) = f'(g(x))g'(x)$

$k'(1) = f'(g(1))g'(1) = f'(2)g'(1) = 5(2) = 10$

3. The US population on July 1 of 2010 was 309.33 million. The population was 311.59 million on July 1 of 2011.

(a) Find an exponential model $p(t)$ to fit this data. Let $t = 0$ on July 1, 2010.

$P_0 = 309.33$

$P(1) = 311.59 = 309.33a^1$

$a = \frac{311.59}{309.33} \approx 1.007306$

$P(t) = 309.33(1.007306)^t$

(b) Use your model to estimate the US population on November 1 of 2013.

$P(10/3) = 309.33(1.007306)^{10/3} \approx 316.9276$

(c) Find $p'(3)$. Interpret the meaning of this number, including units.

$p'(3)$ represents how fast the population is growing on July 1, 2013 in millions/year.

$p'(3) = 309.33(1.007306)^3 \ln(1.007306) \approx 2.3$ mil/year