Practice Exam 3 Solutions

(1) $f(t) = t^2 + 2t$ meters per second is velocity on interval $0 \le t \le 6$.
$n = 3$ subintervals so $\Delta t = 2$

Left Riemann Sum $= f(0)\Delta t + f(2)\Delta t + f(4)\Delta t$

$= 0 + 8\cdot 2 + 24\cdot 2 = 64$ meters

Right Riemann Sum $= f(2)\Delta t + f(4)\Delta t + f(6)\Delta t$

$= 8\cdot 2 + 24\cdot 2 + 48\cdot 2 = 160$ meters.

Left Riemann Sum is underestimate since $f(t)$ is increasing and thus Right Riemann Sum is an overestimate.
Also since $64 < 160$.

Units of $\int_0^6 f(t)\,dt$ are meters and represents the total change in distance of the car in the first 6 seconds.

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(2) I don't have your words so I can't answer this for you, but you can expect this question on the exam.

(3) $g(t)=1-t^2$ for $0 \le t \le 8$.

$g(t)$ is decreasing so the left Riemann Sum is an overestimate.

When $n=4$ subintervals, $\Delta t = 2$.

Left Riemann Sum $= g(0)\Delta t + g(2)\Delta t + g(4)\Delta t + g(6)\Delta t$

$= 1\cdot 2 + (-3)\cdot 2 + (-15)\cdot 2 + (-35)\cdot 2$

$= -104.$

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(4) $\int_a^b f(x)\,dx$

(5) $\int_0^1 (\sqrt{x}-x^2)\,dx$

(6) First find endpoints. $f(t)=0$ when $4-t^2=0 \Rightarrow t=\pm 2$.

Area $= \int_{-2}^{2} (4-t^2)\,dt$.

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(7) t = days $f(t)$ = Kg/day

$\int_5^{15} f(t)\,dt = 4000$ means that between days 5 and 15 4000 Kg of pollution were removed from the lake.

(8) $\int_0^{60} g(t)\,dt$

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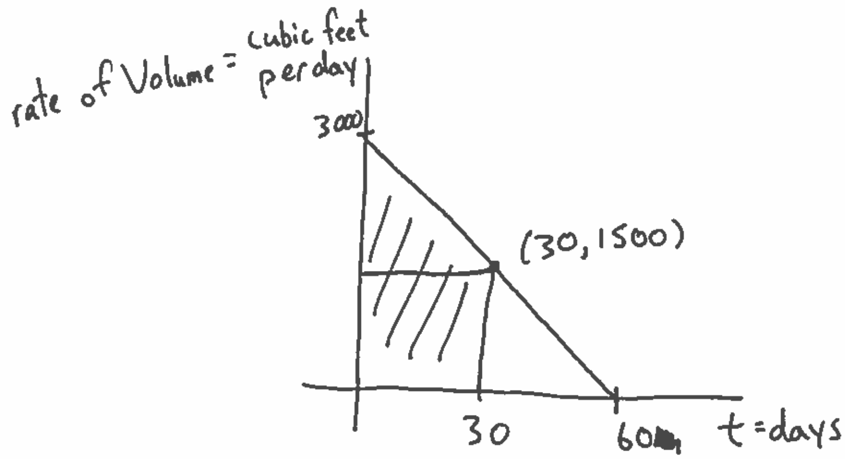
(9) $g(t) = 3000 - 50t$ cubic feet per day. Company charge \$5 a day for each 10 cubic feet.



Total Cubic Feet Used in the first 30 days $= \int_0^{30} (3000 - 50t)\,dt$

= Area of Shaded Region

= Area of triangle + Area of rectangle

$= \frac{1}{2}(30)(1500) + (30)(1500)$

$= 67{,}500$ cubic feet.

Thus Total amount the Company pays is

$$\$5 \cdot \frac{67{,}500}{10} = \$33{,}750.$$

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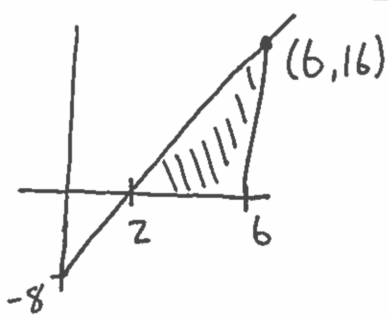
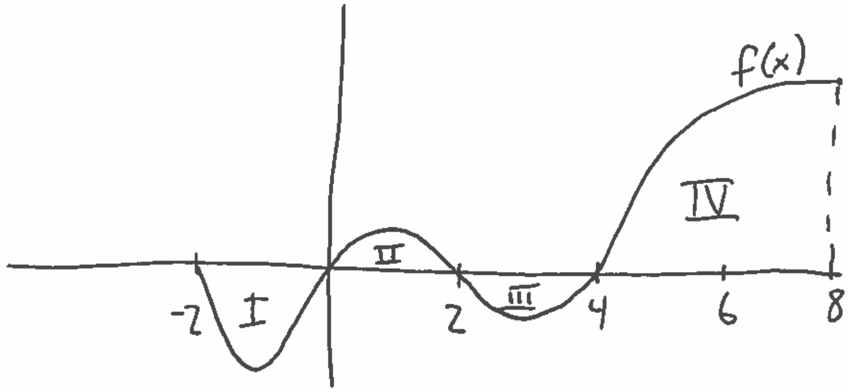
(10) $g(x) = 4x - 8$



$\int_2^6 g(x)\,dx =$ Area of Shaded region

$= \frac{1}{2} 4 \cdot 16 = 32.$

(11)



(a) $\int_{-2}^{8} f(x)\,dx > 0$ because (Area of II + IV) > (Area I + III)

(b) $\int_{-2}^{2} f(x)\,dx < 0$ because (Area of I) > (Area of II)

(c) $\int_{0}^{4} f(x)\,dx = 0$ because (Area of II) = (Area of III)

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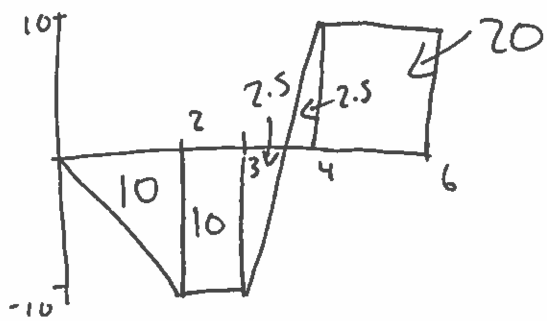
(12)



x	0	2	4	6
f(x)	8	-2	-12	8

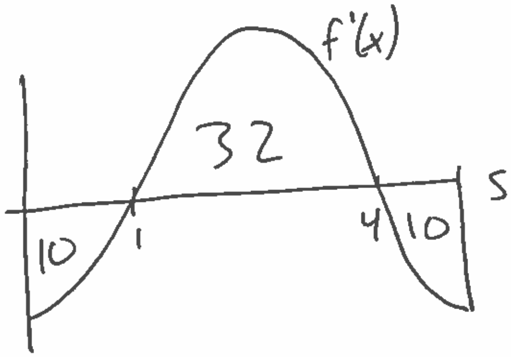
$f(0) = 8$

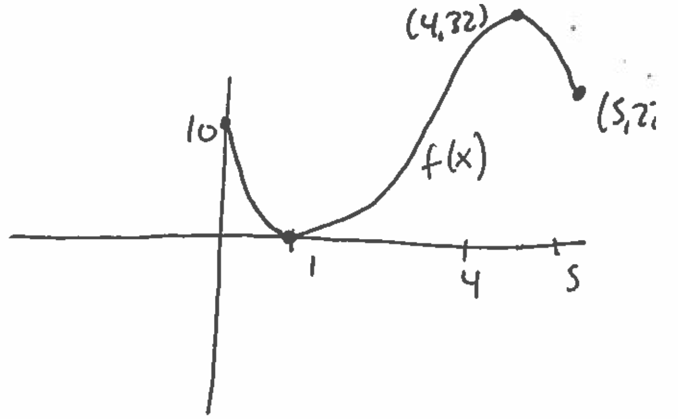
$f(2) = 8 - 10 = -2$

$f(4) = -2 - 10 - 2.5 + 2.5 = -12$

$f(6) = -12 + 20 = 8$

(13)





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(14) $\int(5x+7)dx = \frac{5}{2}x^2 + 7x + C$

(15) $\int(t^2+5t+1)dt = \frac{t^3}{3} + \frac{5}{3}t^2 + t + C$

(16) $\int\left(\frac{3}{x} - \frac{3}{x^2}\right)dx = 3\ln|x| + 3x^{-1} + C$

(17) $\int(3\sqrt{w})dw = \int(3w^{1/2})dw = 3\cdot\frac{w^{3/2}}{3/2} = 2w^{3/2} + C$

(18) $\int\left(e^x + \frac{1}{\sqrt{x}}\right)dx = \int e^x dx + \int x^{-1/2}dx = e^x + \frac{x^{1/2}}{1/2} + C = e^x + 2\sqrt{x} + C.$

(19) $\int(100e^{4t})dt = 100\int e^{4t}dt = 100\cdot\frac{e^{4t}}{4} = 25e^{4t} + C.$

(20) $\int(2\pi r)dr = 2\pi\int r\,dr = 2\pi\left(\frac{r^2}{2}\right) = \pi r^2 + C.$

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(21) $f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}$ (Chain Rule)

(22) $\int_0^6 (2xe^{x^2})dx = e^{6^2} - e^{0^2} = e^{36} - 1.$

(22) $\boxed{\text{Again}}$ $g(t)=t^2\ln(t) \Rightarrow g'(t)=2t\ln(t)+\frac{t^2}{t}$ (Product Rule)

$$=2t\ln(t)+t.$$

(23) $\int_1^4 (2t\ln(t)+t)\,dt = 4^2\ln(4)-1^2\ln(1)=16\ln(4).$

(24) $\int_0^3 t^3\,dt = \frac{t^4}{4}\Big]_0^3 = \frac{3^4}{4} = \frac{81}{4}.$

(25) $\int_4^9 \sqrt{x}\,dx = \frac{x^{3/2}}{3/2}\Big]_4^9 = \frac{2}{3}\left(9^{3/2}-4^{3/2}\right)=\frac{2}{3}(27-8)=\frac{38}{3}.$

(26) $\int_0^2 (3t^2+4t+3)\,dt = t^3+2t^2+3t\Big]_0^2 = (8+8+6)-(0+0+0)$

$$=22.$$

(27) $\int_0^1 2e^x\,dx = 2e^x\Big]_0^1 = 2e^1-2e^0 = 2e-2.$

(28) $\int_2^7 \left(\frac{1}{t}-\frac{2}{t^3}\right)dt = \ln|t| - 2\frac{t^{-2}}{-2}\Big]_2^7 = \left(\ln 7+\frac{1}{7^2}\right)-\left(\ln 2+\frac{1}{2^2}\right)$

(29) $\int_0^1 (y^2+y^4)\,dy = \frac{y^3}{3}+\frac{y^5}{5}\Big]_0^1 = \left(\frac{1}{3}+\frac{1}{5}\right)-(0+0)=\frac{8}{15}.$