

# Sols

## Chapter 3 Differentiation Worksheet

### Basic Rules for Differentiation

- (1)  $\frac{d}{dx}(cf(x)) = cf'(x)$
- (2)  $\frac{d}{dx}(f \pm g(x)) = f'(x) \pm g'(x)$
- (3)  $\frac{d}{dx}(x^n) = nx^{n-1}$
- (4)  $\frac{d}{dx}(e^{kx}) = ke^{kx}$
- (5)  $\frac{d}{dx}(a^x) = \ln(a)a^x$
- (6)  $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

**Exercises:** Find the derivative of the given function unless otherwise specified.  
Assume that  $a, b, c, k$  are constants.

1.  $y = 3t^4$       2.  $h(\theta) = \frac{1}{\sqrt[3]{\theta}} = \theta^{-1/3}$       3.  $Q = aP^2 + bP^3$

4.  $w = 3ab^2q$       5.  $y = ce^t + k \ln t$       6.  $P = 50e^{-0.06t}$

7.  $w = 2^x + \frac{2}{x^3}$       8.  $R(q) = q^2 + 2 \ln(q)$       9.  $P(t) = be^t$

1)  $y' = 12t^3$       2)  $h'(\theta) = -\frac{1}{3} \theta^{-4/3}$       3)  $Q' = 2aP + 3bP^2$

4)  $w' = 3ab^2$       5)  $y' = ce^t + \frac{k}{t}$       6)  $P' = -3e^{-0.06t}$

7)  $\frac{dw}{dx} = \ln(2)2^x - 6x^{-4}$       8)  $R'(q) = 2q + \frac{2}{q}$       9)  $P'(t) = be^t$

10. Find the equation to the line tangent to the graph of  $f(x) = 2x^3 - 5x^2 + 3x - 5$  at  $x = 1$ .

$$f'(x) = 6x^2 - 10x + 3$$

$$f(1) = 2(1)^3 - 5(1)^2 + 3(1) - 5 = -5$$

$$f'(1) = m = 6(1)^2 - 10(1) + 3 = -1$$

$$\boxed{y + 5 = -1(x - 1)}$$

11. Find the equation to the line tangent to the graph of  $y = \ln x$  at  $x = 1$  and use it to approximate  $\ln(1.1)$ .

$$\frac{dy}{dx} = \frac{1}{x}, \quad \left. \frac{dy}{dx} \right|_{x=1} = 1, \quad \ln(1) = 0$$

$$\begin{aligned} \text{So, } f(1.1) &\approx f'(1)(1.1 - 1) + f(1) \\ &= 1(-1) + 0 \\ &= 0.1 \end{aligned}$$

---

**Chain Rule:** For functions  $y = f(z)$  and  $z = g(t)$  the derivative of  $y = f(g(t))$  is given by

$$\frac{d}{dt}(f(g(t))) = f'(g(t)) \cdot g'(t)$$

or

$$\frac{dy}{dt} = \frac{dy}{dz} \cdot \frac{dz}{dt}$$

Combining this rule with the previous rules, if  $z$  is a differentiable function of  $t$  then

$$\frac{d}{dt}(z^n) = nz^{n-1} \frac{dz}{dt}, \quad \frac{d}{dt}(e^z) = e^z \frac{dz}{dt}, \quad \frac{d}{dt}(\ln z) = \frac{1}{z} \frac{dz}{dt}$$


---

**Exercises:** Find the derivative of the given function unless otherwise specified.

12.  $e^{-x^2}$       13.  $(x^2 + 4)^3$       14.  $5 \ln(2t^2 + 3)$   
 15.  $f(\theta) = (e^\theta + e^{-\theta})^{-1}$       16.  $f(x) = \ln(1 - e^{-x})$       17.  $\sqrt{2 + \sqrt{x}}$   
 18.  $j = \ln(\ln(x))$       19.  $P = \sqrt{1 + \ln(x)}$       20.  $y = \sqrt[4]{s^3 + 1}$

$$12) (e^{-x^2})' = -2xe^{-x^2} \quad 13) ((x^2+4)^3)' = 6x(x^2+4)^2 \quad 14) \frac{20t}{2t^2+3}$$

$$15) f'(\theta) = -(e^\theta + e^{-\theta})^{-2} (e^\theta - e^{-\theta}) \quad 16) f'(x) = \frac{e^{-x}}{1 - e^{-x}}$$

$$17) \frac{1}{2} (2+x^{1/2})^{-1/2} (\frac{1}{2} x^{-1/2}) = \frac{1}{4} x^{-1/2} (2+x^{1/2})^{-1/2} \quad 18) \frac{dj}{dx} = \frac{1}{x \ln x}$$

$$19) \frac{dP}{dx} = \frac{1}{2} (1+\ln x)^{-1/2} (\frac{1}{x}) \quad 20) \frac{dy}{ds} = \frac{1}{4} (s^3+1)^{-3/4} (3s^2)$$

21. If you invest  $P$  dollars in a bank account at an annual interest rate of  $r\%$ , then after  $t$  years you will have  $B$  dollars, where

$$B = P\left(1 + \frac{r}{100}\right)^t.$$

- (a) Find  $dB/dt$ , assuming  $P$  and  $r$  are constant. In terms of money, what does  $dB/dt$  represent?
- (b) Find  $dB/dr$ , assuming  $P$  and  $t$  are constant. In terms of money, what does  $dB/dr$  represent?

(a)  $\frac{dB}{dt} = P \ln\left(1 + \frac{r}{100}\right) \left(1 + \frac{r}{100}\right)^t \frac{\text{dollars}}{\text{year}}$  How fast your money is growing.

(b)  $\frac{dB}{dr} = Pt \left(1 + \frac{r}{100}\right)^{t-1} \left(\frac{1}{100}\right) \frac{\text{dollars}}{\text{percent}}$  How much faster your money would grow at a different interest rate.

22. Given  $y = f(x)$  with  $f(1) = 4$  and  $f'(1) = 3$ , find

(a)  $g'(1)$  if  $g(x) = \sqrt{f(x)}$ .

(b)  $h'(1)$  if  $h(x) = f(\sqrt{x})$ .

(a)  $g'(1) = \frac{1}{2} (f(1))^{-1/2} f'(1) = \frac{1}{2} (4)^{-1/2} (3) = \frac{1}{2\sqrt{4}} (3) = \frac{3}{4}$

(b)  $h'(1) = f'(\sqrt{1}) \left(\frac{1}{2} (1)^{-1/2}\right) = f'(1) \left(\frac{1}{2}\right) = 3 \left(\frac{1}{2}\right) = \frac{3}{2}$

**Product and Quotient Rules:** For functions  $u = f(x)$  and  $v = g(x)$  we have

$$(fg)' = f'g + fg' \quad \text{and} \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

or

$$\frac{d(uv)}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx} \quad \text{and} \quad \frac{d(u/v)}{dx} = \frac{\frac{du}{dx} \cdot v - u \cdot \frac{dv}{dx}}{v^2}.$$

**Exercises:** Find the derivative of the given function unless otherwise specified. Assume  $a, b$  are constants.

$$\begin{array}{lll}
 23. f(t) = te^{-2t} & 24. t^2 \ln t & 25. y = (t^2 + 3)e^t \\
 26. w = x \cdot 2^x & 27. z = (3t + 1)(5t + 2) & 28. w = \frac{3y+y^2}{5+y} \\
 29. y = \frac{1+z}{\ln z} & 30. f(x) = axe^{-bx} & 31. g(\alpha) = e^{\alpha e^{-2\alpha}}
 \end{array}$$

$$23) f'(t) = e^{-2t} - 2te^{-2t} \quad 24) 2t \ln(t) + t \quad 25) y' = 2te^t + (t^2 + 3)e^t$$

$$26) \frac{dw}{dx} = 2^x + \ln(2) \times 2^x \quad 27) \frac{dz}{dt} = 3(5t+2) + 5(3t+1)$$

$$28) \frac{dw}{dy} = \frac{(3+2y)(5+y) - (3y+y^2)}{(5+y)^2} \quad 29) \frac{dy}{dz} = \frac{\ln z - (1+z)^{\frac{1}{2}}}{(\ln z)^2}$$

$$30) f'(x) = ae^{-bx} - abxe^{-bx} \quad 31) g'(\alpha) = e^{\alpha e^{-2\alpha}} (e^{-2\alpha} - 2\alpha e^{-2\alpha})$$

32. If a person is lost in the wilderness, the search and rescue team identifies the boundaries of the search area and then uses probabilities to help optimize the chances of finding the person, assuming the subject is immobile. The probability,  $O$ , of the person being outside the search area after the search has begun and the person has not been found is given by

$$O = \frac{I}{1 - (1 - I)E},$$

where  $I$  is the probability of the person being outside the search area at the start of the search and  $E$  is the search effort, a measure of how well the search area has been covered by the resources in the field.

- (a) If there was a 20% chance that the subject was not in the search area at the start of the search, and the search effort was 80%, what is the current probability of the person being outside the search area? (Probabilities are between 0 and 1, so 20%=0.2 and 80%=0.8.)
- (b) In practical terms, what does  $I = 1$  mean? Is this realistic?
- (c) Evaluate  $O'(E)$ . Is it positive or negative? What does that tell you about  $O$  as  $E$  increases?

(a) 55.56% (b) If  $I=1$ , then there is a 100% chance the person is outside the search area. Not realistic.

$$(c) O'(E) = \frac{I(1-I)}{(1-(1-I)E)^2} > 0 \quad \text{Positive rate of change}$$

Thus, as search effort increases, the probability the person is outside the search area also increases.