Intro Math Problem Solving
November 28

The "Lights Out!" Game

A MATLAB version of Lights Out

Reference

Marlow Anderson, Todd Feil,

Turning Lights Out With Linear Algebra,

Mathematics Magazine,

Volume 71, Number 4, October 1998, pages 300-303.

Oscar Martin-Sanchez,

Two Analyses of Lights Out,

Mathematics Magazine,

Volume 74, Number 4, October 2001, pages 295-304.

https://en.wikipedia.org/wiki/Lights_Out_(game)

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"Lights Out!"

The Lights Out Advertisement

https://www.youtube.com/watch?v=rkLyDeygOYc

A Program Plan

At a basic level, a "Lights Out" program needs to:

* initialize data

* display the current board

* get user input (the button choice)
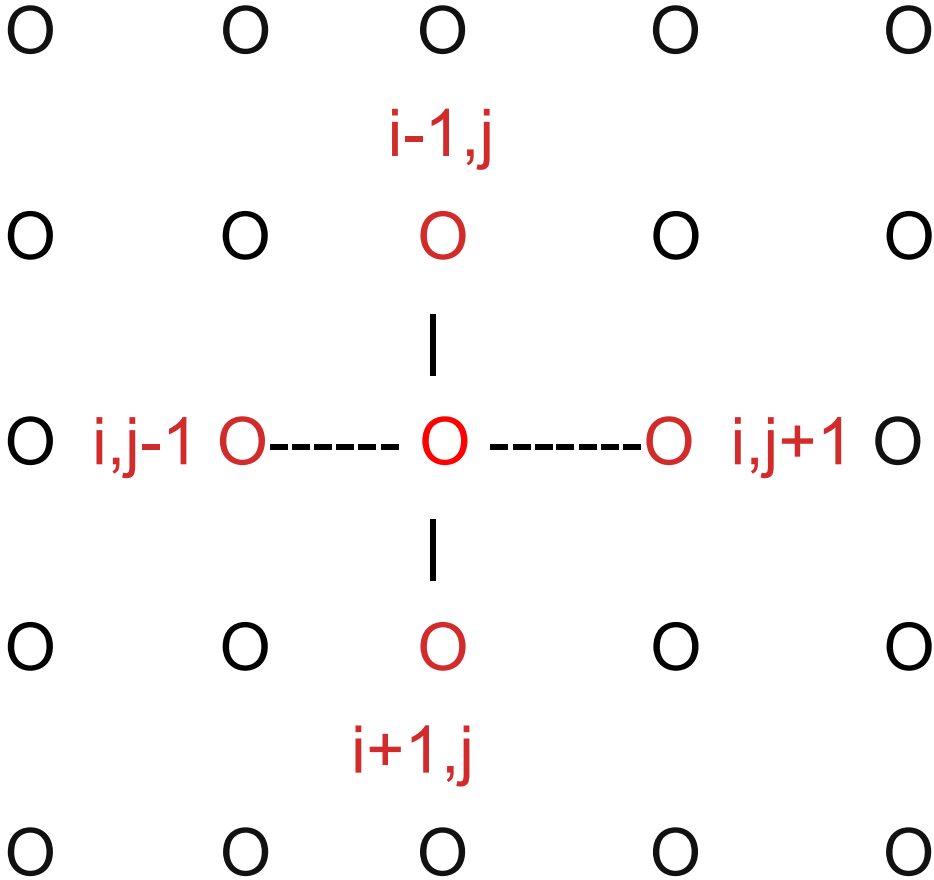
* decide whether to continue or stop

A Puzzle Game

Lights Out is a hand-held electronic game that was release in 1995 by Tiger Electronics.

The game has a 5x5 grid of buttons that light up or turn off when pressed. The game starts with a random configuration of lights turned on, and the user's task is to turn them all off.

However, when you press any button, it switches the state of the light but also the 4 immediate neighboring lights (above, below, left and right.)

The goal is to switch off all the lights in as few moves as possible, but this is not easy!

The "Plus Sign" Neighborhood

O   O   O   O   O

$i-1,j$

O   O   O   O   O

|

O $i,j-1$ O - - - - - O - - - - - - O $i,j+1$ O

|

O   O   O   O   O

$i+1,j$

O   O   O   O   O

Difficulties

When the button being pressed is in the interior, then a total of 5 lights will be switched. If it is along an edge, this will be 4 lights, while a button in the corner affects only 3 lights.

The puzzle is difficult because pushing one button makes multiple things happen, and these events are "switches", that is, some lighted things go off, and some off things come on.

After a few practices, it's not hard to turn off most of the lights, but when you're down to just a few, suddenly it becomes very difficult to see how to proceed without **increasing** the number of lights that are lit.

Is There a Solution?

Actually, it is possible to set up the puzzle in such a way that there is no solution.

However, the version of the game that we will see always has a guaranteed solution.

That's because the puzzle is initialized by starting with all the lights out, and then pressing a number of buttons at random.

A solution is guaranteed: if the player presses those same buttons, in any order, the lights will all be out.

"Brute Force" Strategy

Note that if a button is pushed twice, it is the same as not pushing it at all.

So a solution to the puzzle can be described as a list of which buttons have to be pushed once.

There are 25 buttons, so for each button we have 2 choices, and therefore the number of possible plays is 2^(25)=33,554,432.

If we were very patient, and kept good records, we could solve the puzzle by trying each of the 33 million different configurations of button pushes.

But we are not that patient!

A "Half"-Solution

After pushing random buttons on the puzzle for a while, you might decide that it's time to look for a systematic step-by-step strategy.

It might seem reasonable to start by clearing the lights from the top row. This is actually easy to do: if light (1,J) is ON, then press light (2,J). This will clear row 1.

Step 2: Clear row 2 the same way.

Steps 3 and 4: Clear rows 3 and 4 the same way.

Step 5: We seem to be stuck. We don't have a row 6 that would let us switch off the lights in row 5.

However, this half-solution tells us an important fact: in a sense, the lights in rows 1 through 4 don't matter. We just need a strategy to deal with the last row.

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