

Sample Exam III
Math 166, Section A2

1. The series $27 - 18 + 12 - 8 + \frac{16}{3} - \frac{32}{9} \dots$ converges.

State a reason (with details) for why this series converges (there are two different reasons you could give).

What value does this series converge to?

2. State the integral test; that is, what is it, when can we use it, and what does it tell us?

Use the integral test to explain why the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge.

3. Explain why the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

The first three terms of the series add up to $\frac{5}{6}$. Give a bound on how far the exact value of the series is from $\frac{5}{6}$.

Why do we say the alternating harmonic series only converges *conditionally*?

4. Let us compute the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$.

Use the Ratio Test to determine r , the radius of convergence of this series.

Does the series converge at the left endpoint, $x = -r$, of the interval of convergence?

At the right endpoint, $x = +r$?

Therefore, what is the interval of convergence for this series?

5. What is the Taylor Polynomial $p_3(x)$, of degree 3, about the point $x=1$, that approximates $\ln(x)$?

Write the formula for the *Taylor remainder*, which measures the difference between $p_3(x)$ and $\ln(x)$, at any point x .

6. Consider the curve whose parametric form is $x = t^2 + 4t, y = \sin(2t)$.

What is a formula for the *horizontal velocity*, $\frac{dx}{dt}$?

For $\frac{dy}{dt}$?

What is a formula for the geometric *slope*, $\frac{dy}{dx}$?

At what values of t does this curve have vertical tangent lines?

At what values of t does this curve have horizontal tangent lines?

7. Set up the integral for the arclength, from $t=-1$ to $t=+1$, of the curve whose parametric form is $x = t^2, y = 2t$.

8. Consider the curve $r = 2 + 3 \sin(\theta)$.

When does this curve have vertical tangent lines?

When does this curve have horizontal tangent lines?

(Your answers may simply be a condition that θ must satisfy.)

9. What is the area contained within the polar curve $r = 1 - \sin(\theta)$?