1. If \( f(x) \) is a function, what does it mean to say that \( \lim_{x \to a} f(x) = c \)? Write in English as well as using formal logic (i.e. the symbols \( \forall \) and \( \exists \)). Please don’t write down the answer while looking at your notes! (It is important to know this from memory.)

2. What does it mean to say that \( \lim_{x \to a} f(x) \neq c \)? Your answer should start with “There exists” or “There is”. Write in English as well as using formal logical notation.

3. What does it mean to say that \( \lim_{x \to a} f(x) \) does not exist? Again, answer both in English and in formal notation.

4. Prove that \( \lim_{x \to 3} x^2 = 9 \).

5. Prove that \( \lim_{x \to 4} x + 5 \neq 7 \).

6. Prove that \( \lim_{x \to 0} x \sin x = 0 \).

7. Prove that \( \lim_{x \to 0} \cos(1/x) \) does not exist.

Additional:

3.2, 2, 17, 19.


3.4, 22, 24.

Bonus: (up to 4 points)
Find a variety of examples of valid and invalid arguments in nonmathematical sources such as: newspapers (op-ed columnists are great), magazines, blogs, political speeches and debates, or any other source not specifically dealing with mathematics or logic. Your answers may be a little bit subjective and open to interpretation; if so, please explain.

Formalize the arguments using truth variables and/or predicates, and explain if appropriate. I would be especially interested to see good examples of invalid arguments! (Remember that false premises don’t make an argument invalid.)

Please cite your sources (informally; no formal citation or bibliography is asked for, just say where they came from).

Good examples to look for are: modus ponens and modus tollens; a conditional is equivalent to its contrapositive; converse or inverse error; valid or invalid negation of quantifiers.