Proposition 7. Given $AB$, and straight lines $AC$ and $BC$, we cannot also have straight lines $AD$ and $BD$ with $BC$ and $D$ on the same side of $AB$, and $AD = AC$ and $BD = BC$.

Proof. Suppose we have such straight lines. Then $AC = AD$ and $BC = BD$. Connect $CD$.

We have

$\angle ACD > \angle BCD$ (whole greater than part)

$= \angle BDC$ (pons asinorum — Prop. 5)

$> \angle ADC$ (whole $> \angle$ part)

$= \angle ACD$ (pons asinorum)

So $\angle ACD > \angle ACD$ which is absurd.

So such lines cannot exist.

Note. We assumed $D$ was outside $\triangle ABC$ and to the right of $C$, as Euclid did.
Given a triangle ABC with D as shown.
Bisect AC at E, and extend BE to BF with BE = EF.
(Uses Props 10 and 3)

Then \( \angle AEB = \angle CEF \) by vertical angles (Prop. 15).
and \( AE = CE, BE = EF \),
so \( \triangle ABE \cong \triangle CFE \) by SAS (Prop 4).
So \( \angle BAE = \angle ECF \).

But \( \angle ECD > \angle ECF \), so \( \angle ECD > \angle BAE \).

Similarly we can show \( \angle ECD > \angle ABC \) by bisecting BC instead.
17. Extend BC to D.

By Prop. 16, 
\[ \angle ACD \text{ is bigger than } \angle ABC \text{ or } \angle BAC. \]
Adding \( \angle BCA \) we see
\[ \angle ACD + \angle BCA > \angle ABC + \angle BCA \]
\[ \angle ACD + \angle BCA > \angle BAC + \angle BCA \]
The left side is \( 180^\circ \).
So the sum of \( \angle BCA \) and either of the other two angles is less than \( 180^\circ \). We can extend one of the other two sides to prove that the sum of \( \angle B \) and \( \angle A \) is also \( < 180^\circ \).

19. This uses Prop. 16.

Suppose \( \angle ABC > \angle BCA \).
We claim \( AC > AB \).
If not, \( AC = AB \) or \( AC < AB \). The former contradicts the pons asinorum (Prop. 5). If \( AC < AB \), then by Prop. 18 \( \angle B < \angle C \) which is contrary to what we supposed.
So \( \square \) \( AC > AB \).
26. Given \( \triangle ABC \) and \( \triangle DEF \) with \( \angle ABC = \angle DEF \), \( \angle BCA = \angle EFD \), and \( BC = EF \). (we're proving ASA)

Claim, \( \triangle ABC \cong \triangle DEF \).

If \( AB = DE \) then we're done by SAS (Prop. 4).

Otherwise, without loss of generality \( AB > DE \).

Find \( G \) on \( AB \) so \( BG = DE \).

By SAS (Prop 4) \( \triangle GBC \cong \triangle DEF \) so

\( \angle GCB \preceq \angle DFE \).

But \( \angle GCB < \angle ACB \) (part is less than the whole)

which was assumed equal to \( \angle DFE \)

which is a contradiction.

So \( AB = DE \) and \( \triangle ABC \cong \triangle DEF \).
1. Draw an equilateral triangle $ABC$ on $AB$ as in Euclid's Prop 1.
   Let $CD$ be the 1 bisector.
   Now let $CE$ be the angle bisector of $\angle ACD$.
   $\angle ACE = 15^\circ$.

2. Given $AB = 1$. Draw another ray from $A$ and mark off three equal segments $AC, CD, DE$.
   Draw a circle around $D$ of radius $EB$
   and around $B$ of radius $DE$.
   These intersect in a point $F$.
   $\overline{AF}$ intersects $AB$ in a point $G$.
   Then $BG = \frac{1}{3} AB = \frac{1}{3}$. 
Let $O$ be the center.
Draw any diameter $AB$.
Draw the line to $AB$ at $O$.
This is a diameter $CD$.
$ACBD$ is an inscribed square.

Draw any segment $AB$.
Draw a half circle around $A$ and a full circle around $B$, both of length $AB$.
The circles intersect at $C$ and $D$.
Duplicate the length $AC$ around the circle. Connect the dots to get a regular hexagon.