1. Let $n$ be a positive natural number, $A$ be an $n \times n$ matrix over a field $F$, and $T \in L(F^n)$. In each case determine if $A$ or $T$, respectively, is invertible, not invertible, or there is not sufficient information to decide. Justify your answer.
   a. $T^k = 0$ for some $k \geq 2$.
   b. $AB = 0$ for some nonzero $n \times p$ matrix $B$ with $p \geq 1$.
   c. $A$ is similar to an invertible $n \times n$ matrix $B$.
   d. $\text{nullity}(T) > \text{rank}(T)$.

2. Suppose $V$ is an $n$-dimensional vector space, $n > 0$, and $T \in L(V)$. Let $v$ be a non-zero vector in $V$. Explain why $\alpha = \langle v, Tv, T^2v, \ldots, T^n v \rangle$ must be dependent, and why $\text{span}(\alpha)$ must be $T$-invariant.