Problem 4. Prove that every algebraically closed field of prime characteristic is infinite.

Problem 5. Let $R$ be a commutative ring and let $I$ be a finitely generated nontrivial ideal of $R$. Prove that $R$ has an ideal $M$ such that each of the following properties holds:

i. $I$ is not a subset of $M$, and

ii. for all ideals $J$ of $R$, if $M \subseteq J$ and $M \neq J$, then $I \subseteq J$.

Problem 6. Prove that there is a polynomial $f(x) \in \mathbb{R}[x]$ such that

(a) $f(x) - 1$ belongs to the ideal $(x^2 - 2x + 1)$;
(b) $f(x) - 2$ belongs to the ideal $(x + 1)$, and
(c) $f(x) - 3$ belongs to the ideal $(x^2 - 9)$.

Problem 7. Let the field $E$ be an extension of the field $F$ so that $[E : F]$ is finite. Let $f(x) \in F[x]$ be irreducible and of degree $p$ where $p$ is a prime number. Prove that if $f(x)$ is not irreducible in $E[x]$, then $p$ divides $[E : F]$.