Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. [page 112, #20] Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a $TK_5$ and a $TK_{3,3}$ subgraph.

2. [page 112, #22] A graph is called outplanar if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither $K_4$ nor $K_{2,3}$ as a minor.

3. [page 114, #37] Let $G, G^*$ be dual plan graphs. Prove the following statements:
   1. If $G$ is 2-connected, then $G^*$ is 2-connected.
   2. If $G$ is 3-connected, then $G^*$ is 3-connected.
   3. If $G$ is 4-connected, then $G^*$ need not be 4-connected.

4. [page 140, #13] Show that every critical $k$-chromatic graph is $(k - 1)$-edge-connected.

5. [page 140, #24] For every $k$, find a 2-chromatic graph whose choice number is at least $k$.

6. [page 140, #13] Prove that the choice number of $K_2^r$ is $r$. (Here $K_2^r$ is the complete $r$-partite graph with each part of size 2.)