A lumberjack cuts out a wedge-shaped piece $W$ of a cylindrical tree of radius $r$ obtained by making two saw cuts to the tree's center, one horizontally and one at an angle $\theta$. Compute the volume of the wedge $W$ using Cavalieri's principle. (See Figure 5.1.12.)

**Figure 5.1.12** Find the volume of $W$.

**Quiz:**

The solution will be posted later today.

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The volume $V$ of the slice that is drawn above is

$$\frac{1}{2} \cdot b \cdot h \cdot t$$

where $t = \Delta x$, $h = b \tan \theta$, and $b = \sqrt{r^2 - x^2}$

The volume of the slice is

$$\frac{1}{2} bh \Delta x = \frac{1}{2} b \tan \theta \Delta x = \frac{1}{2} (r^2 - x^2) \tan \theta \Delta x$$

The volume of the wedge is obtained by adding up the volume of the slices and taking the limit

$$V = \frac{1}{2} \tan \theta (r^2 - x^3) \bigg|_0^r$$

$$V = \tan \theta (r^3 - \frac{r^3}{3}) = \frac{2}{3} (\tan \theta) r^3$$

Thus, the volume of the wedge is

$$\frac{2}{3} (\tan \theta) r^3$$