Let \( X \) and \( Y \) be sets and \( A \subset X \) and \( B \subset Y \). Let \( f: X \to Y \) be a function from \( X \) to \( Y \).

So we have:

\[
f: \quad X & \quad \to \quad Y \\
\cup & \quad \cup \\
A & \quad B .
\]

Define the **image of \( A \) under \( f \)**, denoted by \( f[A] \), to be the set

\[
f[A] := \{ f(x) \in Y : x \in A \} \subset Y .
\]

Define the **inverse image of \( B \)**, denoted by \( f^{-1}[B] \), to be the set

\[
f^{-1}[B] := \{ x \in X : f(x) \in B \} \subset X .
\]

Thus, by definition of \( f[A] \),

\[
y \in f[A] \iff \exists a \in A \text{ s.t. } f(a) = y .
\]

Also, by definition of \( f^{-1}[B] \),

\[
x \in f^{-1}[B] \iff f(x) \in B .
\]

**Proposition 1.** Let \( A_1, A_2, A_i \subset X \) and \( B_1, B_2, B_i \subset Y \) for \( i \) in an indexing set \( I \). Then

\[
A_1 \subset A_2 \implies f[A_1] \subset f[A_2] \quad (1) \\
B_1 \subset B_2 \implies f^{-1}[B_1] \subset f^{-1}[B_2] \quad (2)
\]

and

\[
A \subset f^{-1}[f[A]] \quad (3) \\
f[f^{-1}[B]] \subset B \quad (4)
\]

and

\[
f[\cup_{i \in I} A_i] = \cup_{i \in I} f[A_i] \quad (5) \\
f[\cap_{i \in I} A_i] \subset \cap_{i \in I} f[A_i] \quad (6)
\]

and

\[
f^{-1}[\cup_{i \in I} B_i] = \cup_{i \in I} f^{-1}[B_i] \quad (7) \\
f^{-1}[\cap_{i \in I} B_i] = \cap_{i \in I} f^{-1}[B_i] . \quad (8)
\]

Furthermore,

- if \( f \) is injective (i.e., one-to-one), the set equality holds in (3)
- if \( f \) is surjective (i.e., onto), then set equality holds in (4).

To help remember (6), think about what can happen when \( \cap_{i \in I} A_i = \emptyset \).