INSTRUCTIONS

(1) The mark box above indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(2) You may not use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) For the do by hand problems, to receive credit you must:
   (a) work in a logical fashion, show all your work, indicate your reasoning:
       no credit will be given for an answer that just appears;
       such explanations help with partial credit
   (b) if a line/box is provided, then:
       — show you work BELOW the line/box
       — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer.

(5) For the multiple choice problems, please.
   • First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
   • Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
   • Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(6) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(7) This exam covers (from Calculus by Stewart, 6th ed., ET):
   § 11.9–11.11 and 6.1–6.3.

NAME: __________________________

PIN: ______________________________
0. Fill in: the boxes in problem 0A and the lines in problem 0B.

0A. Taylor/Maclaurin Polynomials and Series

Let \( y = f(x) \) be a function with derivatives of all orders in an interval \( I \) containing \( x_0 \).
Let \( y = P_N(x) \) be the \( N^{\text{th}} \)-order Taylor polynomial of \( y = f(x) \) about \( x_0 \).
Let \( y = R_N(x) \) be the \( N^{\text{th}} \)-order Taylor remainder of \( y = f(x) \) about \( x_0 \).
Let \( y = P_\infty(x) \) be the Taylor series of \( y = f(x) \) about \( x_0 \).
Let \( c_n \) be the \( n^{\text{th}} \) Taylor coefficient of \( y = f(x) \) about \( x_0 \).

In open form (i.e., with \( \ldots \) and without a \( \sum \)-sign)
\[
P_N(x) = \]

In closed form (i.e., with a \( \sum \)-sign and without \( \ldots \))
\[
P_N(x) = \]

In open form (i.e., with \( \ldots \) and without a \( \sum \)-sign)
\[
P_\infty(x) = \]

In closed form (i.e., with a \( \sum \)-sign and without \( \ldots \))
\[
P_\infty(x) = \]

We know that \( f(x) = P_N(x) + R_N(x) \). Taylor’s BIG Theorem tells us that, for each \( x \in I \),
\[
R_N(x) = \quad \text{for some } c \text{ between } \quad \quad \quad \quad \quad \quad \quad \text{and} \quad \quad \quad \quad \quad \quad \quad \}

The formula for \( c_n \) is
\[
c_n = \]

A Maclaurin series is a Taylor series about the center \( x_0 = \) .
0B. Volume of Revolutions. Let’s say we revolve some region in the $xy$-plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

- In parts a, fill in the blanks with: $x$ or $y$.
- In parts b and c, fill in the blanks with a formula involving some of: $2$, $\pi$, radius, radius$_{big}$, radius$_{little}$, average radius, height, and/or thickness.

▶ Disk/Washer Method. Let’s find the volume of this solid of revolution using the disk or washer method.

a. If the axis of revolution is:
   - the $x$-axis, or parallel to the $x$-axis, then we partition the $z$-axis.
   - the $y$-axis, or parallel to the $y$-axis, then we partition the $z$-axis.

b. If we use the **disk method**, then the volume of a typical disk is:

   
   [Formula]

   If we use the **washer method**, then the volume of a typical washer is:

   
   [Formula]

c. If we partition the $z$-axis, where $z$ is either $x$ or $y$, the $\Delta z$ is the [Formula]

▶ Shell Method. Let’s find the volume of this solid of revolution using the shell method.

a. If the axis of revolution is:
   - the $x$-axis, or parallel to the $x$-axis, then we partition the $z$-axis.
   - the $y$-axis, or parallel to the $y$-axis, then we partition the $z$-axis.

b. If we use the **shell method**, then the volume of a typical shell is:

   
   [Formula]

c. If we partition the $z$-axis, where $z$ is either $x$ or $y$, the $\Delta z$ is the [Formula]

1a. Express $e^{-x^2}$ as an infinite series, in closed form (i.e., with $\sum$ sign and no ...).

$$e^{-x^2} = \sum_{n=0}^{\infty}$$

1b. Express $\int e^{-x^2} \, dx$ as an infinite series, in closed form (i.e., with $\sum$ sign and no ...).

$$\int e^{-x^2} \, dx = C + \sum_{n=0}^{\infty}$$

1c. Approximate $\int_{0}^{1} e^{-x^2} \, dx$ by a finite sum (you need to do the arithmetic since you do not have a calculator on hand, just leave your answer as a sum of a finite number of numbers) where the accuracy (i.e. error) is less than or equal to $\frac{1}{11(5!)}$. Hint: Alternating Series Remainder test.

$$\int_{0}^{1} e^{-x^2} \, dx \approx$$
TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 2 – 11

Instructions.
- Indicate (by circling) your solution to each problem.
- You may choose up to 3 answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 6 points. For a problem with precisely two answers marked, one of which is correct, 3 points. For a problem with precisely three answers marked, one of which is correct, 1 point. All other cases, 0 points.

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<tr>
<th>PROBLEM</th>
<th>Your Solutions</th>
<th>points</th>
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INSTRUCTIONS for MULTIPLE CHOICE PROBLEMS

- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with problems 1–10, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- Select at most one response for each problem.
- The scoring is: 8 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

2. Using a known (commonly used) Taylor series, find the Taylor series for \( f(x) = x \cos(4x) \) about the center \( x_0 = 0 \).
   a. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{n!} \]
   b. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!} \]
   c. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!} \]
   d. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{2n+1}}{(2n+1)!} \]
   e. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{2n+1}}{(2n+1)!} \]

3. Using a known (commonly used) Taylor series, evaluate \( \int \tan^{-1}(t^2) \, dt \) as a power series.
   a. \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(2n+1)(4n+3)} \]
   b. \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(2n+1)(4n+1)} \]
   c. \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+2}}{(2n+1)(2n+3)} \]
   d. \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+2}}{(2n+1)} \]
   e. \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+3}}{(2n+1)(2n+3)} \]

4. Find the 2nd order Taylor polynomial for \( f(x) = \sqrt{x} \) about the center \( x_0 = 8 \).
   a. \[ 2 + \frac{x}{12} - \frac{x^2}{9} \]
   b. \[ 2 + \frac{(x-8)}{12} + \frac{(x-8)^2}{9} \]
   c. \[ 2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9} \]
   d. \[ 2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9} \]
   e. none of these

5. Find the Taylor series for \( f(x) = (1 - 5x)^{-3} \) about the center \( x_0 = 0 \).
   a. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 5^n (n+1)(n+2)}{2} x^n \]
   b. \[ \sum_{n=0}^{\infty} \frac{5^n (n+1)(n+2)}{2} x^n \]
   c. \[ \sum_{n=0}^{\infty} (-1)^{n+1} \frac{5^n}{n!} x^n \]
   d. \[ \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n \]
   e. none of these

6. Consider the function \( f(x) = e^x \) over the interval \((-1, 3)\). The 4th order Taylor polynomial of \( y = f(x) \) about the center \( x_0 = 0 \) is

\[ P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{4} \frac{x^n}{n!} \]

The 4th order Remainder term \( R_4(x) \) is defined by \( R_4(x) = f(x) - P_4(x) \) and so \( e^x \approx P_4(x) \) where the approximation is within an error of \( |R_4(x)| \). Using Taylor’s (BIG) Theorem, find a good upper bound for \( |R_4(x)| \) that is valid for each \( x \in (-1, 3) \).

   a. \( \frac{(e^{-1})(3^4)}{4!} \)
   b. \( \frac{(e^3)(3^4)}{4!} \)
   c. \( \frac{(e^{-1})(3^4)}{4!} \)
   d. \( \frac{(e^3)(3^4)}{5!} \)
   e. none of these
7. Express the area of the region enclosed by \( y = x^2 \) and \( y = 4x - x^2 \) as an integral.
   a. \( \int_0^4 [(4x - x^2) - x^2] \, dx \)  
   b. \( \int_0^4 [x^2 - (4x - x^2)] \, dx \)  
   c. \( \int_0^2 [(4x - x^2) - x^2] \, dx \)  
   d. \( \int_0^2 [x^2 - (4x - x^2)] \, dx \)  
   e. none of these

   Problem Source: § 6.1 Exercise # 12.

8. Let \( R \) be the region bounded by the curves 
   \[ y = 1 - x^2 \quad \text{and} \quad y = 0. \]
   Express as an integral the volume of the solid generated by revolving \( R \) about the \( x \)-axis.
   a. \( \pi \int_0^1 \sqrt{1 - y} \, dy \)  
   b. \( 2\pi \int_0^1 y \sqrt{1 - y} \, dy \)  
   c. \( \pi \int_0^1 (1 - x^2)^2 \, dx \)  
   d. \( \pi \int_{-1}^1 (1 - x^2)^2 \, dx \)  
   e. none of these

   Problem Source: § 6.2 Exercise # 2.

9. Let \( R \) be the region bounded by the curves 
   \[ y = \frac{x^2}{4} \quad \text{and} \quad y = 5 - x^2. \]
   Express as integral(s) the volume of the solid generated by revolving \( R \) about the \( x \)-axis.
   a. \( \pi \int_{-2}^2 [(5 - x^2) - \left(\frac{x^2}{4}\right)]^2 \, dx \)  
   b. \( \pi \int_{-2}^2 \left[(5 - x^2)^2 - \left(\frac{x^2}{4}\right)^2\right] \, dx \)  
   c. \( 2\pi \int_0^5 2y \sqrt{5 - y} \, dy \)  
   d. \( 2\pi \int_1^5 y \sqrt{4y} \, dy \)  
   e. none of these

   Problem Source: § 6.2 Exercise # 8.

10. Let \( R \) be the region bounded by the curves 
    \[ y = \frac{1}{x} \quad \text{and} \quad y = 0 \quad \text{and} \quad x = 1 \quad \text{and} \quad x = 2. \]
    Express as integral(s) the volume of the solid generated by revolving \( R \) about the \( y \)-axis.
    a. \( 2\pi \int_{\frac{1}{2}}^1 \frac{1}{x} \, dx \)  
    b. \( 2\pi \int_1^2 x \, dx \)  
    c. \( 2\pi \int_1^2 1 \, dx \)  
    d. \( \pi \int_0^1 \left[(\frac{1}{y})^2 - (1)^2\right] \, dy \)  
    e. none of these

    Problem Source: § 6.3 Exercise # 3.

11. Let \( R \) be the region bounded by the curves 
    \[ y = x \quad \text{and} \quad y = 4x - x^2. \]
    Express as integral(s) the volume of the solid generated by revolving \( R \) about the line \( x = 7. \)
    a. \( 2\pi \int_0^3 x \left[x - (4x - x^2)\right] \, dx \)  
    b. \( 2\pi \int_0^3 x \left[(4x - x^2) - x\right] \, dx \)  
    c. \( 2\pi \int_0^3 (7 - x) \left[x - (4x - x^2)\right] \, dx \)  
    d. \( 2\pi \int_0^3 (7 - x) \left[(4x - x^2) - x\right] \, dx \)  
    e. none of these

    Problem Source: § 6.3 Exercise # 22.