INSTRUCTIONS

(1) The **MARK BOX** above indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(2) You may **not** use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) For Problems 1–4, to receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning:**

   *no credit will be given for an answer that *just appears*;*

   such explanations help with partial credit

   (b) if a line/box is provided, then:

   — show you work BELOW the line/box

   — put your answer on/in the line/box

   (c) if no such line/box is provided, then box your answer.

(5) Problems 5–11 are multiple choice.
   • First, indicate (to yourself) your answers directly on the **STATEMENT OF MULTIPLE CHOICE PROBLEMS** part.
   • Once finished with the multiple choice problems, go back to the **HAND IN PART** and indicate your answers on the table provided.
   • Hand in the **HAND IN PART**. You can take the **STATEMENT OF MULTIPLE CHOICE PROBLEMS** part home with you (so you can check your answers once the solutions are posted).

(6) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(7) This exam covers (from *Calculus* by Stewart, 6th ed., ET):

7.1–7.5, 7.8, 11.1.
0. Fill in the blanks (each worth 1 point).

a. \( \int \frac{du}{a} = \) \[ \text{______} \] \(|u| + C \)

b. If \( a \) is a constant and \( a > 0 \) but \( a \neq 1 \), then \( \int a^u \, du = \) \[ \text{______} \] \(+ C \)

c. \( \int \cos u \, du = \) \[ \text{______} \] \(+ C \)

d. \( \int \sec^2 u \, du = \) \[ \text{______} \] \(+ C \)

e. \( \int \sec u \tan u \, du = \) \[ \text{______} \] \(+ C \)

f. \( \int \sin u \, du = \) \[ \text{______} \] \(+ C \)

g. \( \int \csc^2 u \, du = \) \[ \text{______} \] \(+ C \)

h. \( \int \sec u \cot u \, du = \) \[ \text{______} \] \(+ C \)

i. \( \int \tan u \, du = \) \[ \text{______} \] \(+ C \)

j. \( \int \cot u \, du = \) \[ \text{______} \] \(+ C \)

k. \( \int \sec u \, du = \) \[ \text{______} \] \(+ C \)

l. \( \int \csc u \, du = \) \[ \text{______} \] \(+ C \)

m. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{\sqrt{a^2 - u^2}} \, du = \) \[ \text{______} \] \(+ C \)

n. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{a^2 + u^2} \, du = \) \[ \text{______} \] \(+ C \)

o. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{u \sqrt{u^2 - a^2}} \, du = \) \[ \text{______} \] \(+ C \)

p. Partial Fraction Decomposition. If one wants to integrate \( \frac{f(x)}{g(x)} \) where \( f \) and \( g \) are polynomials and \([\text{degree of } f] \geq [\text{degree of } g] \), then one must first do \[ \text{______} \]

q. Integration by parts formula: \( \int u \, dv = \) \[ \text{______} \]

r. Trig substitution: (recall that the integrand is the function you are integrating)
   if the integrand involves \( a^2 - u^2 \), then one makes the substitution \( u = \) \[ \text{______} \]

s. Trig substitution:
   if the integrand involves \( a^2 + u^2 \), then one makes the substitution \( u = \) \[ \text{______} \]

t. Trig substitution:
   if the integrand involves \( u^2 - a^2 \), then one makes the substitution \( u = \) \[ \text{______} \]

u. trig formula ... your answer should involve trig functions of \( \theta \), and not of \( 2\theta \): \( \sin(2\theta) = \) \[ \text{______} \]

v. trig formula ... \( \cos(2\theta) \) should appear in the numerator: \( \cos^2(\theta) = \) \[ \text{______} \]

w. trig formula ... \( \cos(2\theta) \) should appear in the numerator: \( \sin^2(\theta) = \) \[ \text{______} \]

x. trig formula ... since \( \cos^2 \theta + \sin^2 \theta = 1 \), we know that the corresponding relationship between tangent (i.e., tan) and secant (i.e., sec) is \[ \text{______} \]

y. \( \text{arctan}(-1) = \) \[ \text{______} \] RADIANS. (your answer should be an angle)
1. \[
\int (\sin x)(\sec x) \, dx = \quad + C
\]
\[ \int x^2 e^x \, dx = + C \]
3. \[
\int \frac{\sqrt{9 - x^2}}{x^2} \, dx = + C
\]
4. For the following SEQUENCES:
   - if the limit exists, find it
   - if the limit does not exist, then say that it DNE.

   Put your ANSWER IN the box and show your WORK BELOW the box.

   4a. \[ \lim_{n \to \infty} \frac{5n^2 + 4\sqrt{n}}{6n^2 + 7n + 1} = \]

   4b. \[ \lim_{n \to \infty} \frac{-5n^8 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \]

   4c. \[ \lim_{n \to \infty} \frac{5n^3 + 4\sqrt{n}}{6n^8 + 7n^2 + 1} = \]
**TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 5 – 11**

**Instructions.**

- Indicate (by circling, boxing, or x-ing) your solution to each problem.
- You may choose up to 3 answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 5 points. For a problem with precisely two answers marked, one of which is correct, 3 points. For a problem with precisely three answers marked, one of which is correct, 1 point. All other cases, 0 points.

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<th>PROBLEM</th>
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Your Solutions
Instructions.

• Indicate (by circling, boxing, or x-ing) your solution to each problem on the Hand In Part of the exam.

• You may choose up to 3 answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 5 points. For a problem with precisely two answers marked, one of which is correct, 3 points. For a problem with precisely three answers marked, one of which is correct, 1 point. All other cases, 0 points.

5. Evaluate the integral
\[ \int_0^1 \frac{x}{x^2 + 9} \, dx \]
a. \( \frac{1}{2} (\ln 10 - \ln 9) \)  
b. \( \frac{1}{2} (\ln 1 - \ln 0) \)  
c. \( (\ln 10 - \ln 9) \)  
d. \( (\ln 1 - \ln 0) \)  
e. None of these

6. Evaluate the integral
\[ \int_0^4 \frac{x}{x + 9} \, dx . \]
Hint: do you have (strictly) bigger bottoms?
a. \( 4 - 9 \ln(13) + 9 \ln(9) \)  
b. \( 13 - 9 \ln(4) + \ln(3) \)  
c. \( \frac{1}{9 \ln(13)} - \ln(3) \)  
d. \( 4 - 13 \ln(9) + 3 \ln(18) \)  
e. None of these

7. Evaluate \( \int_1^0 x^p \, dx \) for different values of \( p \). Which of the following integrals is equal to \( \frac{1}{25} \)?
a. \( \int_0^1 x^{0.2} \, dx \)  
b. \( \int_0^1 x^{0.5} \, dx \)  
c. \( \int_0^1 x^{0.7} \, dx \)  
d. \( \int_0^1 x^2 \, dx \)  
e. \( \int_0^1 x^{2.5} \, dx \)

8. Evaluate the integral
\[ \int_0^\frac{\pi}{2} \sin^2 x \cos^3 x \, dx . \]
a. \( -\frac{2}{15} \)  
b. \( \frac{2}{15} \)  
c. \( -\frac{4}{7} \)  
d. \( \frac{4}{7} \)  
e. none of these

9. Evaluate the integral
\[ \int \frac{dx}{(x^2 + 2x + 2)^2} . \]
Hint: complete the square: \( x^2 + 2x + 2 = (x + ?)^2 + ?. \)
a. \( \frac{1}{2} \left( \arctan(x + 1) + \frac{x+1}{x^2+2x+2} \right) + C \)  
b. \( \frac{1}{2} \left( \tan(x + 1) + \frac{1}{x^2+2x+2} \right) + C \)  
c. \( \frac{1}{2} \left( \tan(x + 1) + \frac{x+1}{x^2+2x+2} \right) + C \)  
d. \( \frac{1}{2} \left( \arctan(x + 1) + \frac{1}{x^2+2x+2} \right) + C \)  
e. none of these

10. Evaluate the integral
\[ \int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} \, dx . \]
Hint: Do we have (strictly) bigger bottoms?
Recall: \( (\ln a) + (\ln b) = \ln(ab) \) and \( (\ln a) - (\ln b) = \ln(\frac{a}{b}) \)
a. \( \ln(6) + \frac{7}{6} \)  
b. \( \ln(6) + \frac{1}{6} \)  
c. \( \ln(\frac{7}{3}) + \frac{7}{6} \)  
d. \( \ln(\frac{7}{3}) + \frac{1}{6} \)  
e. none of these

11. Compute
\[ \lim_{n \to \infty} \left( -\frac{1}{2} \right)^n . \]
a. 0  
b. 1  
c. diverges to infinity  
d. does not exist but does not diverges to infinity  
e. none of these