MARK BOX

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<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
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NAME: ___________________________

PIN: ___________________________

INSTRUCTIONS:

(1) To receive credit you must:
   (a) work in a logical fashion, show all your work, indicate your reasoning;
       no credit will be given for an answer that just appears;
       such explanations help with partial credit
   (b) if a line/box is provided, then:
       — show you work BELOW the line/box
       — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points.
    Check that your copy of the exam has all of the problems.
(3) You may not use a calculator, books, personal notes.
(4) During this exam, do not leave your seat unless you have permission. If you have a question,
    raise your hand. When you finish: turn your exam over, put your pencil down, and raise
    your hand.
(5) If you do not make at least 17.5 out of the 35 points on Problem 1, then your score for the
    entire exam will be whatever you made on Problem 1.
(6) This exam covers (from Calculus by Stewart, 6\textsuperscript{th} ed., ET):
    11.2–11.8

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain
the University of South Carolina’s Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the
University’s Honor Code then I will receive a failing grade for this course and be referred to the academic
Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature: ___________________________
1. Fill-in-the-blanks/boxes. All series \( \sum \) are understood to be \( \sum_{n=1}^{\infty} \).

1a. \textit{n-th-term test} for an arbitrary series \( \sum a_n \).
   If \( \lim_{n \to \infty} a_n \neq 0 \) or \( \lim_{n \to \infty} a_n \) does not exist, then \( \sum a_n \) ________________.

1b. \textit{Geometric Series} where \( -\infty < r < \infty \). The series \( \sum r^n \)
   - converges if and only if \( |r| \) ________________
   - diverges if and only if \( |r| \) ________________

1c. \textit{p-series} where \( 0 < p < \infty \). The series \( \sum \frac{1}{n^p} \)
   - converges if and only if \( p \) ________________
   - diverges if and only if \( p \) ________________

1d. \textit{Integral Test} for a positive-termed series \( \sum a_n \) where \( a_n \geq 0 \).
   Let \( f: [1, \infty) \to \mathbb{R} \) be so that
   - \( a_n = f(\square) \) for each \( n \in \mathbb{N} \)
   - \( f \) is a ________________ function
   - \( f \) is a ________________ function
   - \( f \) is a ________________ function.
   Then \( \sum a_n \) converges if and only if ________________ converges.

1e. \textit{Comparison Test} for a positive-termed series \( \sum a_n \) where \( a_n \geq 0 \). (Fill in the blanks with \( a_n \) and/or \( b_n \).
   - If \( 0 \leq a_n \leq b_n \) for all \( n \in \mathbb{N} \) and \( \sum b_n \) converge, then \( \sum a_n \) converge.
   - If \( 0 \leq b_n \leq a_n \) for all \( n \in \mathbb{N} \) and \( \sum b_n \) diverge, then \( \sum a_n \) diverge.

1f. \textit{Limit Comparison Test} for a positive-termed series \( \sum a_n \) where \( a_n \geq 0 \).
   Let \( b_n > 0 \) and \( \lim_{n \to \infty} \frac{a_n}{b_n} = L \).
   If \( \square < L < \square \), then \( \sum a_n \) converges if and only if \( \square \).

1g. \textit{Ratio and Root Tests} for arbitrary-termed series \( \sum a_n \) with \( -\infty < a_n < \infty \).
   Let \( \rho = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) or \( \rho = \lim_{n \to \infty} |a_n|^{\frac{1}{n}} \).
   - If \( \rho \) ________________ then \( \sum a_n \) converges absolutely.
   - If \( \rho \) ________________ then \( \sum a_n \) diverges.
   - If \( \rho \) ________________ then the test is inconclusive.

1h. \textit{Alternating Series Test} for an alternating series \( \sum (-1)^n a_n \) where \( a_n > 0 \) for each \( n \in \mathbb{N} \).
   If
   - \( a_n \) ________________ \( a_{n+1} \) for each \( n \in \mathbb{N} \)
   - \( \lim_{n \to \infty} a_n = \square \)
   then \( \sum (-1)^n a_n \) ________________
1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ __________
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ ________ and $\sum |a_n|$ ________
- $\sum a_n$ is divergent if and only if $\sum a_n$ ________

1j. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=1}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

1k. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

T F If $\sum a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

T F If $\lim_{n \to \infty} a_n = 0$, then $\sum a_n$ converges

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.

T F If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $S_N = \sum_{n=1}^{N} r^n$, then $S_N = \frac{r - r^N}{1 - r}$, for when $r \neq 1$ since we don’t like to divide by zero.
2. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \]

- [ ] absolutely convergent
- [x] conditionally convergent
- [ ] divergent
3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

\[ \sum_{n=1}^{\infty} \frac{4n^7 - n^6 + 21}{11n^8 - n + 17} \]

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

You may use, without showing, that for \( n \geq 1 \)

\[ a_n := \frac{4n^7 - n^6 + 21}{11n^8 - n + 17} > 0 . \]

Hint. What would be easier: CT or LCT?
4. Let

\[ a_n = \frac{n!}{(2n - 1)!} \]

4a. Find an expression for \( \frac{a_{n+1}}{a_n} \) that does NOT have a factorial sign (that is a ! sign) in it.

\[ \frac{a_{n+1}}{a_n} = \]

4b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

\[ \sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n - 1)!} \]

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent
5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

\[ \sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{3n^2 + 8n - 1} \]

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent
6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

\[
\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}
\]

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent
7. Geometric Series. (On this page, you should do basic algebra but you do NOT have to do any fancy arithmetic (eg, just making up numbers, you can leave \( \int \frac{17}{18} \) as just that.) Let, for \( N \geq 51 \),

\[
 s_N = \sum_{n=51}^{N} 2^{\frac{3^n+1}{5^n}}.
\]

7a. Do some algebra to write \( s_N \) as \( \sum_{n=51}^{N} c \cdot r^n \) for an appropriate constant \( c \) and ratio \( r \).

\[
 s_N = \sum_{n=51}^{N}
\]

7b. Using the method from class (rather than some formula), find an expression for \( s_N \) in closed form (i.e. without a summation \( \sum \) sign nor some dots \ldots ).

\[
 s_N =
\]

7c. Does \( \sum_{n=51}^{\infty} 2^{\frac{3^n+1}{5^n}} \) converge or diverge? If it converges, find its sum. Justify your answer.

\[
 \sum_{n=51}^{\infty} 2^{\frac{3^n+1}{5^n}}
\]