INSTRUCTIONS:

(1) To receive credit you must:
   (a) \textbf{work in a logical fashion, show all your work, indicate your reasoning;} no credit will be given for an answer that just appears:
   - such explanations help with partial credit
   (b) if a line/box is provided, then:
   - show you work BELOW the line/box
   - put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer

(2) The \textbf{MARK BOX} indicates the problems along with their points.
    Check that your copy of the exam has all of the problems.

(3) You may \textbf{not} use an electronic device, a calculator, books, personal notes.

(4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) If you do not make at least 12.5 out of 25 points on Problem 1, then your score for the entire exam will be whatever you made on Problem 1.

(6) This exam covers (from Calculus (ET) by Stewart 6th ed.):
   Sections 7.1 – 7.5, 7.8, 11.1.

Hints:

(1) You can check your answers to the indefinite integrals by differentiating.

(2) For more partial credit, box your $u - du$ substitutions.

\textbf{Honor Code Statement}

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature: ______________________
1. Fill in the blanks (each worth 1 point).

1a. \( \int \frac{du}{u} = \ln |u| + C \)

1b. If \( a \) is a constant and \( a > 0 \) but \( a \neq 1 \), then \( \int a^u \, du = \frac{a}{\ln a} u + C \)

1c. \( \int \cos u \, du = -\sin u + C \)

1d. \( \int \sec^2 u \, du = \tan u + C \)

1e. \( \int \sec u \tan u \, du = \sec u + C \)

1f. \( \int \sin u \, du = -\cos u + C \)

1g. \( \int \csc^2 u \, du = -\cot u + C \)

1h. \( \int \csc u \cot u \, du = -\csc u + C \)

1i. \( \int \tan u \, du = \ln |\cos u| + C = \ln |\sec u| + C \)

1j. \( \int \cot u \, du = \ln |\sin u| + C = -\ln |\csc u| + C \)

1k. \( \int \sec u \, du = \ln |\sec u + \tan u| + C = -\ln |\sec u - \tan u| + C \)

1l. \( \int \csc u \, du = -\ln |\csc u + \cot u| + C = \ln |\csc u - \cot u| + C \)

1m. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{a\sqrt{a^2 - u^2}} \, du = \frac{\sin^{-1} u}{a} + C \)

1n. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{a^2 + u^2} \, du = \frac{\tan^{-1} u}{a} + C \)

1o. If \( a \) is a constant and \( a > 0 \) then \( \int \frac{1}{a^2 - u^2} \, du = \frac{\sec^{-1} u}{a} + C \)

1p. Partial Fraction Decomposition. If one wants to integrate \( \frac{f(x)}{g(x)} \) where \( f \) and \( g \) are polynomials and \( \deg(f) \geq \deg(g) \), then one must first do \( \text{long division} \)

1q. Integration by parts formula: \( \int u \, dv = uv - \int v \, du \)

1r. Trig substitution: (recall that the integrand is the function you are integrating)
   - if the integrand involves \( a^2 - u^2 \), then one makes the substitution \( u = a \sin \theta \)

1s. Trig substitution:
   - if the integrand involves \( a^2 + u^2 \), then one makes the substitution \( u = a \tan \theta \)

1t. Trig substitution:
   - if the integrand involves \( u^2 - a^2 \), then one makes the substitution \( u = a \sec \theta \)

1u. trig formula ... your answer should involve trig functions of \( \theta \), and not of \( 2\theta \): \( \sin(2\theta) = 2 \sin \theta \cos \theta \)

1v. trig formula ... \( \cos(2\theta) \) should appear in the numerator: \( \cos^2(\theta) = \frac{1}{2} \left( 1 + \cos 2\theta \right) \)

1w. trig formula ... \( \cos(2\theta) \) should appear in the numerator: \( \sin^2(\theta) = \frac{1}{2} \left( 1 - \cos 2\theta \right) \)

1x. trig formula ... since \( \cos^2 \theta + \sin^2 \theta = 1 \), we know that the corresponding relationship between tangent (i.e., \( \tan \)) and secant (i.e., \( \sec \)) is \( \tan^2 \theta = \sec^2 \theta - 1 \)

1y. \( \arcsin \left( -\frac{\sqrt{2}}{4} \right) = \frac{-\pi}{4} \) RADIANS. (your answer should be an angle)
\[ \int (\sin x)(\sec x) \, dx = -\ln |\cos x| + C \leq \ln |\sec x| + C + C \]

\[ \int (\sin x)(\sec x) \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{du}{u} \]

\[ u = \cos x \]
\[ du = -\sin x \, dx \]

\[ = -\ln |u| + C = -\ln |\cos x| + C \]

\[ = + \ln (1 + \cos x) + C = \ln \frac{1 + \cos x}{1 + \cos x} + C = \ln |\sec x| + C \]
3. \[
\int x \tan^2 x \, dx = x \tan x - \frac{x^2}{2} + \ln |\cos x| + C
\]
\[ \int \ln(1+x) \, dx = (x+1) \ln(1+x) - x + C \]

Hint: bring to the other side idea.

\[ \int \ln(1+x) \, dx = x \ln(1+x) - \int \frac{x}{1+x} \, dx = x \ln(1+x) - \int \frac{1}{1+x} \, dx = x \ln(1+x) - \ln(1+x) + C = (x+1) \ln(1+x) - x + C \]

\[ u = \ln(1+x) \quad dv = dx \]
\[ du = \frac{1}{1+x} \, dx \quad v = 1+x \]

\[ \int \ln(1+x) \, dx = (1+x) \ln(1+x) - \int \frac{1+x}{1+x} \, dx = (1+x) \ln(1+x) - \int dx = (1+x) \ln(1+x) - x + C \]
\[
\int \frac{x^2}{\sqrt{4-x^2}} \, dx = 2 \sin^{-1} \left( \frac{x}{2} \right) - x \sqrt{4-x^2} + C
\]

\[
\int \frac{\sqrt{4-x^2}}{4-x^2} \, dx \rightarrow \sin \theta = \frac{x}{2}
\]

\[
x = 2 \sin \theta
\]

\[dx = 2 \cos \theta \, d\theta
\]

\[
\begin{align*}
\sqrt{4-x^2} &= \sqrt{4-4 \sin^2 \theta} = 2 \sqrt{1-\sin^2 \theta} = 2 \cos \theta \\
\int \frac{x^2}{\sqrt{4-x^2}} \, dx &= \int \frac{4 \sin^2 \theta}{2 \cos \theta} \, 2 \cos \theta \, d\theta = 4 \int \sin^2 \theta \, d\theta \\
&= 4 \cdot \frac{1}{2} \int (1 - \cos 2\theta) \, d\theta = 2 \int d\theta - 2 \cdot \frac{1}{2} \int (\cos 2\theta) (2 \, d\theta) \\
&= 2\theta - \sin 2\theta + C = 2\theta - 2 \sin \theta \cos \theta + C \\
&= 2 \sin^{-1} \left( \frac{x}{2} \right) - 2 \left( \frac{x}{2} \right) \left( \frac{\sqrt{4-x^2}}{2} \right) + C \\
&= \left[ 2 \sin^{-1} \left( \frac{x}{2} \right) - \frac{x \sqrt{4-x^2}}{2} \right] + C
\]
\[\int \frac{x^4 + 2x + 2}{x^4(x+1)} \, dx = \frac{2x^{-3}}{-3} + \ln |x+1| + C\]

Hint \(x^4 = (x-0)^4\)

\[\frac{x^4 + 2x + 2}{x^4(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+1} \quad \text{Note} \quad x^4 = (x-0)^4\]

\[\Rightarrow \frac{x^4 + 2x + 2}{x^4(x+1)} = \frac{A x^3(x+1) + B x^2(x+1) + C x(x+1) + D(x+1) + E x^4}{x^4(x+1)}\]

\[\Rightarrow x^4 + 2x + 2 = A x^3(x+1) + B x^2(x+1) + C x(x+1) + D(x+1) + E x^4\]

\[
\begin{align*}
\text{At } x = 0 & \Rightarrow 2 = D \\
\text{At } x = -1 & \Rightarrow 1 = E
\end{align*}
\]

- \(x^4 : 1 = A + E \quad \Rightarrow \quad A = 0\)
- \(x^3 : 0 = A + B\) \quad \Rightarrow \quad B = 0\)
- \(x^2 : 0 = B + C\) \quad \Rightarrow \quad C = 0\)
- \(x : 2 = C + D\) \quad \Rightarrow \quad D = 2\)

\[\int \frac{x^4 + 2x + 2}{x^5 + x^4} \, dx = \int \left( \frac{2}{x^4} + \frac{1}{x+1} \right) \, dx\]

\[= \int 2x^{-4} \, dx + \int \frac{dx}{x+1} = \frac{2x^{-3}}{-3} + \ln |x+1| + C\]
\[ \int_1^\infty \frac{1}{(3x+1)^4} \, dx = \frac{1}{576} \]

Warning: write your solution in proper form.

\[ \lim_{c \to \infty} \int_1^c \frac{1}{(3x+1)^4} \, dx = \lim_{c \to \infty} \left[ -\frac{1}{9(3x+1)^3} \right]_1^c \]

\[ = 0 - \left( -\frac{1}{9(3\cdot1)^3} \right) \]

\[ = 0 - \left( -\frac{1}{9\cdot4} \right) \]

\[ = 0 + \frac{1}{576} \]

\[ \int \frac{1}{(3x+1)^4} \, dx = \frac{1}{576} \]

\[ \frac{1}{3} \int u^{-4} \, du \]

\[ \frac{1}{3} \left( \frac{u^{-3}}{-3} \right) + C \]

\[ = -\frac{1}{9(3x+1)^3} + C \]
8. For the following SEQUENCES in 1-5:
   - if the limit exists, find it
   - if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

\[
\lim_{n \to \infty} \frac{5n^2 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = 0
\]

\[
\lim_{n \to \infty} \frac{5n^2 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \lim_{n \to \infty} \frac{\frac{5n^2}{n^3} + \frac{4\sqrt{n}}{n^3}}{\frac{6n^3}{n^3} + \frac{7n^2}{n^3} + \frac{1}{n^3}} = \frac{0 + 0}{6 + 0 + 0} = 0
\]

\[
\lim_{n \to \infty} \frac{5n^2 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \infty \equiv \text{DNE}
\]

\[
\lim_{n \to \infty} \frac{5n^8 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \lim_{n \to \infty} \frac{\frac{5n^8}{n^8} + \frac{4\sqrt{n}}{n^8}}{\frac{6n^3}{n^8} + \frac{7n^2}{n^8} + \frac{1}{n^8}} = \frac{\frac{5}{\infty}}{0} = 0
\]

\[
\lim_{n \to \infty} \frac{5n^3 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \frac{5}{6}
\]

\[
\lim_{n \to \infty} \frac{5n^3 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \lim_{n \to \infty} \frac{\frac{5n^3}{n^3} + \frac{4\sqrt{n}}{n^3}}{\frac{6n^3}{n^3} + \frac{7n^2}{n^3} + \frac{1}{n^3}} = \frac{5 + 0}{6 + 0 + 0} = \frac{5}{6}
\]