INSTRUCTIONS:

(1) To receive credit you must:
   (a) work in a logical fashion, show all your work, indicate your reasoning;
       no credit will be given for an answer that just appears;
       such explanations help with partial credit
   (b) if a line/box is provided, then:
       — show you work BELOW the line/box
       — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points.
    Check that your copy of the exam has all of the problems.
(3) You may not use an electronic device, a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When
    you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from Calculus (ET) by Stewart 6th ed.):
    Sections 7.1 – 7.5, 7.8, 11.1.

Problem Inspiration:
1. You were warned.
2. example in class
3. homework problem § 7.5 # 9
4. homework problem § 7.5 # 15
5. homework problem § 7.5 # 21
6. Handout of 100 integrals # 35
7. homework problem § 7.5 # 41

Hints:
(1) You can check your answers to the indefinite integrals by differentiating.
(2) For more partial credit, box your $u - du$ substitutions.

Honor Code Statement
I understand that it is the responsibility of every member of the Carolina community to uphold and maintain
the University of South Carolina’s Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature: ________________________________
You were warned on the first day of the semester.

1. Fill in the blanks (each worth 1 point).
   a. \( f_{\tan} f_{\tan} \)  
   b. If \( \alpha \) is a constant and \( \alpha > 0 \) but \( \alpha \neq 1 \), then \( f_{\sec} f_{\sec} \)  
   c. \( f_{\sec} f_{\sec} \)  
   d. \( f_{\tan} f_{\tan} \)  
   e. \( f_{\sec} f_{\sec} \)  
   f. \( f_{\sec} f_{\sec} \)  
   g. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   h. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   i. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   j. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   k. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   l. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   m. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   n. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   o. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   p. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   q. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   r. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   s. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   t. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   u. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   v. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   w. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   x. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   y. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   z. If \( \alpha \) is a constant and \( \alpha > 0 \) then \( f_{\sec} f_{\sec} \)  
   
   Integration by parts formula: \( f_{\tan} f_{\tan} \)  
   
   Trig substitution: (recall that the integrand is the function you are integrating) 
   if the integrand involves \( u^2 + u^2 \), then one makes the substitution \( u = \tan \theta \)  
   
   Trig substitution: 
   if the integrand involves \( u^2 - u^2 \), then one makes the substitution \( u = \sin \theta \)  
   
   Trig substitution: 
   if the integrand involves \( u^2 - u^2 \), then one makes the substitution \( u = \sec \theta \)  
   
   trig formula ... your answer should involve trig functions of \( \theta \), and not of \( 2\theta \): \( \sin(2\theta) = 2\sin\theta \cos\theta \)  
   
   trig formula ... your answer should involve \( \cos(2\theta) \) in it: \( \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta)) \)  
   
   trig formula ... your answer should involve \( \cos(2\theta) \) in it: \( \sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)) \)  
   
   trig formula ... since \( \cos^2\theta + \sin^2\theta = 1 \), we know that the corresponding relationship between 
   tangent (i.e., \( \tan \)) and secant (i.e., \( \sec \)) is \( 1 + \tan^2\theta = \sec^2\theta \)  
   
   arctan \((-1)\) = \(-\frac{\pi}{4}\)  
   
   RADIANS. (your answer should be an angle)
\[ \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C \]

Hint: trig formulas from problem 1 come in handy.

\[ \int \sin^2 \ x \ dx = \frac{1}{2} \int (1 - \cos 2x) \ dx \]

\[ = \frac{1}{2} \int 1 \ dx - \frac{1}{2} \int \cos 2x \ dx \]

\[ = \frac{1}{2} \cdot x - \frac{1}{2} \cdot \frac{1}{2} \int \cos 2x (2 \ dx) \]

\[ = \frac{1}{2} \cdot x - \frac{1}{4} \sin (2x) + C \]
3. 
\[ \int x^{3/2} \ln x \, dx = \frac{2 \cdot x^{5/2} \ln x}{5} - \frac{4}{25} x^{3/2} + C \]

\[
 u = \ln x \quad \text{d}v = x^{3/2} \, dx \\
 du = \frac{1}{x} \, dx \quad v = \frac{2}{5} x^{5/2}
\]

\[
\int x^{3/2} \ln x \, dx = \frac{2 \cdot x^{5/2} \ln x}{5} - \frac{2}{5} \int x^{3/2} \cdot x^{-1} \, dx \\
= \frac{2 \cdot x^{5/2} \ln x}{5} - \frac{2}{5} \int x^{3/2} \, dx \\
= \frac{2 \cdot x^{5/2} \ln x}{5} - \frac{2}{5} \cdot \frac{2}{5} x^{5/2} + C
\]
\[ \int \frac{x-1}{x^2+2x} \, dx = \frac{1}{2} \ln |x| + \frac{3}{2} \ln |x+2| + C \]

Hint: \( x^2 + 2x = x(x+2) = (x-0)(x+2) \)

\[ \frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)}{x(x+2)} + \frac{B(x)}{x(x+2)} \]

\[ \Rightarrow x-1 = A(x+2) + B \cdot x \]

\[ \begin{cases} x = 0 & -1 = 2A \\ x = -2 & -3 = -2B \end{cases} \]

\[ A = -\frac{1}{2} \quad B = \frac{3}{2} \]

Or, can equate coefficients:

\[ x : \ 1 = A + B \]

Constant: \(-1 = 2A\)

\[ \int \frac{x-1}{x^2+2x} \, dx = \int \frac{-\frac{1}{2}}{x} \, dx + \int \frac{\frac{3}{2}}{x+2} \, dx \]

\[ = -\frac{1}{2} \ln |x| + \frac{3}{2} \ln |x+2| + C \]

Or:

\[ \ln |x|^{-\frac{1}{2}} + \ln |x+2|^\frac{3}{2} + C \]

Or:

\[ \ln \left[ \frac{|x+2|^\frac{3}{2}}{|x|^\frac{1}{2}} \right] + C \]

Or:

\[ \ln \left[ \frac{1}{|x+2|^2} \right] + C \]

\[ \ln e + C \]
5a. Complete the square. The two lines should have numbers on them. The box should have a plus or minus sign in it.

\[ x^2 - 4x = (x - \frac{2}{2})^2 - 4 \]

\[ \downarrow \]

\[ x - 4x + 4 \]

5b. \[
\int \frac{1}{\sqrt{x^2 - 4x}} \, dx = \ln \left| \frac{x - 2}{2} + \frac{\sqrt{(x-2)^2 - 4}}{2} \right| + C
\]

\[
\int \frac{dx}{\sqrt{x^2 - 4x}} = \int \frac{dx}{\sqrt{(x-2)^2 - 2^2}} = \int \frac{2 \sec \theta \tan \theta \, d\theta}{2 \tan \theta}
\]

\[
= \int \sec \theta \, d\theta
\]

\[
= \ln |\sec \theta + \tan \theta| + C
\]
\[ \int \sec^3 x \tan^3 x \, dx = \] 

\[ \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C \]
\[ \int_1^{\infty} \frac{1}{(2x+1)^3} \, dx = \frac{1}{36} \]

Warning: Write your solution in proper form.

\[ \int_1^{\infty} \frac{dx}{(2x+1)^3} = \lim_{b \to \infty} \left[ \frac{1}{2} \int_{x=1}^{x=b} (2x+1)^{-3} \, (2 \, dx) \right] \]

\[ = \lim_{b \to \infty} \frac{1}{2} \left. \frac{(2x+1)^{-2}}{-2} \right|_{x=1}^{x=b} \]

\[ = -\frac{1}{4} \lim_{b \to \infty} \left. \left( \frac{1}{(2x+1)^2} \right) \right|_{x=1}^{x=b} \]

\[ = -\frac{1}{4} \lim_{b \to \infty} \left[ \frac{1}{(2b+1)^2} - \frac{1}{3^2} \right] \]

\[ = -\frac{1}{4} \left[ 0 - \frac{1}{3^2} \right] \]

\[ = \frac{1}{4} \cdot \frac{1}{9} \]

\[ = \frac{1}{36} \]