MARK BOX

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
<th>in class</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a – y</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2 a – o</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4 – 16 (10 pts each)</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>17 a–f</td>
<td>10</td>
<td>take home</td>
</tr>
<tr>
<td>18 take home</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

NAME: ____________________________

please check the box of your section

Section 005 (WF 8:00 am)

or

Section 006 (WF 9:05 am)

The (whole) final exam covers (from Calculus by Anton, Bivens, Davis 8th ed.):
Sections: 7.1 - 7.4, 7.6, 7.7, 8.1 - 8.5, 8.8, 10.1 - 10.10.
The above MARK BOX and below Problem Inspiration gives you an idea of the break down.

Problem Inspiration:

IN CLASS: 1.–16.

1.–3. Fill in the blanks and True/False (omitting sections 10.7, 10.9, 10.10)
4.&5. Chapter 7
6.–11. Chapter 8
12.–16 Sections 10.1 – 10.6 and 10.8

TAKE HOME 17.–18.

17.&18 Sections 10.7, 10.9, 10.10

INSTRUCTIONS for TAKE HOME PART, which is due at 2pm on December 15, 2006:

(1) Turn in all 5 pages of this exam.
(2) To receive credit you must:
   (a) work in a logical fashion, show all your work, indicate your reasoning;
       no credit will be given for an answer that just appears;
       such explanations help with partial credit
   (b) if a line/box is provided, then:
       — show you work BELOW the line/box
       — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(3) You can use books and notes.
(4) You can not use a calculator. You can not use a computer. Thus you do not need
to do lots of multiplication and may leave you answers as you would on previous exams
(e.g. \( \frac{7(17)}{3} \) is acceptable).
(5) You can not receive help from other people.

SIGNATURE REQUIRED:

I hereby verify that I did NOT receive help from other people on this final exam take-home part.

Signature: ____________________________
Let \( y = f(x) \) be a function with derivatives of all orders in an interval \( I \) containing \( x_0 \).
Let \( y = P_N(x) \) be the \( N \)th-order Taylor polynomial of \( y = f(x) \) about \( x_0 \).
Let \( y = R_N(x) \) be the \( N \)th-order Taylor remainder of \( y = f(x) \) about \( x_0 \).
Let \( y = P_\infty(x) \) be the Taylor series of \( y = f(x) \) about \( x_0 \).
Let \( c_n \) be the \( n \)th Taylor coefficient of \( y = f(x) \) about \( x_0 \).

A. In open form (i.e., with \ldots and without a \( \sum \)-sign)

\[
P_N(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x-x_0)^3 + \cdots + \frac{f^{(N)}(x_0)}{N!}(x-x_0)^N
\]

B. In closed form (i.e., with a \( \sum \)-sign and without \ldots)

\[
P_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n
\]

C. In open form (i.e., with \ldots and without a \( \sum \)-sign)

\[
P_\infty(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \ldots
\]

D. In closed form (i.e., with a \( \sum \)-sign and without \ldots)

\[
P_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n
\]

E. We know that \( f(x) = P_N(x) + R_N(x) \). Taylor’s BIG Theorem tells us that, for each \( x \in I \),

\[
R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!} (x-x_0)^{N+1}
\]

for some \( c \) between \( x \) and \( x_0 \).

F. The formula for \( c_n \) is

\[
c_n = \frac{f^{(n)}(x_0)}{n!}
\]
17. Do parts (a) - (f) for the following:

\[ f(x) = xe^x \quad x_0 = 0 \quad J = (-17, 2) \]

You might find it easier to do problems (a) - (f) in a different order. Just do what you find easiest.

Use only:
- the definition of Taylor polynomial
- the definition of Taylor series
- the theorem/error-estimate on the \( N^{th} \)-Remainder term for Taylor polynomials.

**Do NOT use a known Taylor Series (i.e., do not use methods from section 10.10).**

17a Find the following. Note the 1st column are functions of \( x \) and the 2nd and 3rd columns are numbers (do not get out a calculator and start pushing keys for the numbers).

\[
\begin{array}{cccc}
\hline
f^{(0)}(x) = & f^{(0)}(x_0) = & c_0 = \\
\hline
f^{(1)}(x) = & f^{(1)}(x_0) = & c_1 = \\
\hline
f^{(2)}(x) = & f^{(2)}(x_0) = & c_2 = \\
\hline
f^{(3)}(x) = & f^{(3)}(x_0) = & c_3 = \\
\hline
f^{(4)}(x) = & f^{(4)}(x_0) = & c_4 = \\
\hline
f^{(5)}(x) = & \text{nothing for here} & \text{nothing for here} \\
\hline
\end{array}
\]

17b Find the \( N^{th} \)-order Taylor polynomial of \( y = f(x) \) about \( x_0 \) in OPEN form for \( N = 0, 1, 2, 3, 4 \).

\[
\begin{array}{llll}
\hline
P_0(x) = & \\
\hline
P_1(x) = & \\
\hline
P_2(x) = & \\
\hline
P_3(x) = & \\
\hline
P_4(x) = & \\
\hline
\end{array}
\]
17c. Find the Taylor series of \( y = f(x) \) about \( x_0 \) in OPEN form.

\[
P_\infty(x) =
\]

17d. Find the Taylor series of \( y = f(x) \) about \( x_0 \) in CLOSED form.

\[
P_\infty(x) =
\]

17e. Consider the given interval \( J \). Find an upper bound for the maximum of \( |f^{(5)}(x)| \) on the interval \( J \).

You answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). You answer cannot have an: \( N, x, x_0, c \).

\[
\max_{c \in J} |f^{(5)}(c)| \leq
\]

17f. Consider the given interval \( J \). Using Taylor’s Remainder Theorem (i.e., Taylor’s Big Theorem), find an upper bound for the maximum of \( |R_4(x)| \) on the interval \( J \). You answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). You answer cannot have an: \( N, x, x_0, c \).

\[
\max_{x \in J} |R_4(x)| \leq
\]
Using the fact that
\[
\frac{1}{1 - r} = \sum_{n=0}^{\infty} r^n \quad \text{when} \quad |r| < 1, \quad (\ast)
\]
find a power series expansion of
\[
\frac{x}{4 + 100x^2}
\]
and state when it is valid. Simplify your answer so that your power series has the form
\[
\sum_{n=0}^{\infty} c_n x^\text{some power}
\]
for some constants \(c_n\).

\[
\frac{x}{4 + 100x^2} = \sum_{n=0}^{\infty} \quad \text{valid when} \quad |x| < \]