Warning: there are 5 problems: 2 problems on the front and 3 problems on the back. Turn your paper over

NAME: ________________________________

There are 5 problems. Each problem is worth 2 points.

Please check the box of your section

Section 005 (WF 8:00 am)

or

Section 006 (WF 9:05 am)

INSTRUCTIONS: Indicate your reasoning. Put answers in box and show work below the box.

1. \[ \lim_{n \to \infty} \frac{1}{2^n} = 0 \]

\[ \lim_{n \to \infty} \frac{1}{2^n} = \lim_{n \to \infty} \left( \frac{1}{2} \right)^n = 0 \quad \text{for} \quad r = \frac{1}{2}, \quad |r| < 1 \]

or

\[ \lim_{n \to \infty} \frac{1}{2^n} = \approx = 0. \]

2. \[ \lim_{n \to \infty} (17)^n = \infty \] or diverge or DNE

\[ \lim_{n \to \infty} (17)^n = \infty, \quad r = 17, \quad |r| > 1 \]
3. \[ \lim_{n \to \infty} \frac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} = \frac{5}{17} \]

\[ \lim_{n \to \infty} \frac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} = \lim_{n \to \infty} \frac{5 + \frac{6}{n^2} + \frac{3}{n^3}}{17 + \frac{9}{n} + \frac{4}{n^2}} = \frac{5 + 0 + 0}{17 + 0 + 0} = \frac{5}{17} \]

Divide numerator and denominator by \( n \) (highest power you see) = \( n^3 \)

4. \[ \lim_{n \to \infty} \frac{n^3}{\ln n} \]

\[ \lim_{n \to \infty} \frac{3n^2}{\ln n} = \lim_{n \to \infty} \frac{6n}{18} = \lim_{n \to \infty} \frac{6}{18} = \frac{1}{3} = 0 \]

Helpful intuition: \( y = e^x \) grows so much faster than \( y = x^3 \) that the answer should be zero. We know, for \( 0 < p < \infty \) and \( n \) big enough, \( n^p < e^n \). So how to show the limit is zero. Way #1 is to use l'Hopital's rule. Way #2 is to use the Squeeze Theorem.

\[ 0 \leq \frac{n^3}{e^n} \leq \frac{n^3}{n^4} = \frac{1}{n} \xrightarrow{n \to \infty} 0 \]

5. \[ \lim_{n \to \infty} \frac{\sqrt[3]{n^2 + 5}}{\sqrt[3]{64n^4 + 17n}} = \left( \frac{1}{64} \right)^{\frac{1}{6}} \]

| hint: | \[ \frac{\sqrt[3]{n^2 + 5}}{\sqrt[3]{64n^4 + 17n}} = \frac{(n^2 + 5)^{\frac{1}{3}}}{(64n^4 + 17n)^{\frac{1}{3}}} = \frac{(n^2 + 5)^{\frac{1}{3}}}{(64n^4 + 17n)^{\frac{1}{3}}} \]

\[ \xrightarrow{n \to \infty} \left[ \frac{1 + \frac{10n^2 + 25}{64n^4 + 17n}}{64 + \frac{17}{n^3}} \right] \]

\[ \xrightarrow{n \to \infty} \left[ \frac{1 + 0 + 0}{64 + 0} \right] \]

\[ \xrightarrow{\text{no!}} \]

\[ \frac{\text{num.}}{\text{den.}} \text{ by } n^4 \]

(see above problem 3)

Remark: Unfortunately I saw on too many papers:

\[ (n^2 + 5)^{\frac{1}{3}} = n^{\frac{2}{3}} + 5^{\frac{1}{3}} \]

Eq: \[ \sqrt[n]{a + b} \neq \frac{\sqrt[n]{a} + \sqrt[n]{b}}{2} \]