INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning:**
       no credit will be given for an answer that *just appears*;
       such explanations help with partial credit
   (b) when applicable put your answer on/in the line/box provided
   (c) if no such line/box is provided, then box your answer

(2) The **MARK BOX** indicates the problems along with their points.
    Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish:
    turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
    Sections 6.6, 6.8, 6.9, 7.1–7.4, 7.6, 7.7, 8.1–8.8, 10.1–10.10.

**Problem Inspiration:** homework and old exams.

Solutions will be available on the course homepage later this afternoon.
1. Fill in the blanks (each worth 1 point) and boxes (each worth 2 points).

1a. \( \int \frac{dx}{x} = \boxed{} \quad |x| + C \)

1b. \( D_x e^x = \boxed{} \)

1c. If \( a > 0 \) but \( a \neq 1 \), then \( D_x a^x = \boxed{} \)

*Hint:* \( a^x = e^{\ln(a^x)} = e^{x \ln(a)} \). Your answer should **not** have an “\( e \)” in it.

1d. \( D_x \tan x = \boxed{} \)

1e. \( \int \sec x \tan x = \boxed{} + C \)

1f. Integration by parts formula:
   \( \int u \, dv = \boxed{} \)

1g. Trig substitution: (recall that the *integrand* is the function you are integrating)
   if the integrand involves \( a^2 - u^2 \), then one makes the substitution \( u = \boxed{} \)

1h. Trig substitution:
   if the integrand involves \( a^2 + u^2 \), then one makes the substitution \( u = \boxed{} \)

1i. Partial Fraction Decomposition. If one wants to integrate \( \frac{f(x)}{g(x)} \) where \( f \) and \( g \) are polynomials
   and \( \deg(f) \geq \deg(g) \), then one must first do \( \boxed{} \)

1j. **Integral Test:** \( a_n > 0 \)

Let \( f: [1, \infty) \to \mathbb{R} \) be so that

- \( a_n = f(\boxed{}) \) for each \( n \in \mathbb{N} \)
- \( f \) is a \( \boxed{} \) function
- \( f \) is a \( \boxed{} \) function
- \( f \) is a \( \boxed{} \) function

Then \( \sum a_n \) converges if and only if \( \boxed{} \) converges.

1k. **Comparison Test:** \( a_n > 0 \)

- If \( 0 \leq a_n \leq b_n \) for all \( n \in \mathbb{N} \) and \( \sum b_n \boxed{} \), then \( \sum a_n \boxed{} \).
- If \( 0 \leq b_n \leq a_n \) for all \( n \in \mathbb{N} \) and \( \sum b_n \boxed{} \), then \( \sum a_n \boxed{} \).

1l. **Limit Comparison Test:** \( a_n > 0 \)

Let \( b_n > 0 \) and \( \lim_{n \to \infty} \frac{a_n}{b_n} = L \).

If \( \boxed{} < L < \boxed{} \), then \( \sum a_n \) converges if and only if \( \sum b_n \boxed{} \)
1m. Ratio Test: \( a_n > 0 \)

Let \( \rho = \lim_{n \to \infty} \frac{a_{n+1}}{a_n} \).

- If \( \rho < \underline{\text{________}} \) then \( \sum a_n \) converges.
- If \( \rho > \underline{\text{________}} \) then \( \sum a_n \) diverges.
- If \( \rho = \underline{\text{________}} \) then the test is inconclusive.

1n. Root Test: \( a_n > 0 \)

Let \( \rho = \lim_{n \to \infty} \left( a_n \right)^{\frac{1}{n}} \).

- If \( \rho < \underline{\text{________}} \) then \( \sum a_n \) converges.
- If \( \rho > \underline{\text{________}} \) then \( \sum a_n \) diverges.
- If \( \rho = \underline{\text{________}} \) then the test is inconclusive.

1o. Alternating Series Test: \( a_n > 0 \)

If

- \( a_n \underline{\text{________}} a_{n+1} \) for each \( n \in \mathbb{N} \)
- \( \lim_{n \to \infty} a_n = \underline{\text{________}} \)

then \( \sum (-1)^n a_n \underline{\text{________________________}} \).

1p. \( n^{\text{th}}\)-term test: \( a_n \)'s are arbitrary

If \( \lim_{n \to \infty} a_n \neq 0 \) or \( \lim_{n \to \infty} a_n \) does not exist, then \( \sum a_n \underline{\text{________________________}} \).

1q. Consider the interval \( I = (a - R, a + R) \) center about \( x = a \) and of radius \( R \).

Let \( y = f(x) \) be a function that can be differentiated \( N \) times \( x = a \). Then the \( N^{\text{th}}\)-order Taylor polynomial \( y = P_N(x) \) of \( f \) about \( a \) is (your answer should have a summation sign \( \sum \) in it)

\[
P_N(x) =
\]

1r. Consider the interval \( I = (a - R, a + R) \) center about \( x = a \) and of radius \( R \).

Let \( y = f(x) \) be a function that can be differentieated \( N + 1 \) times for each \( x \in I \).

Consider the the \( N^{\text{th}}\)-order Taylor Reminder term \( R_N(x) \), where \( f(x) = P_N(x) + R_N(x) \).

Then an upper bound for \( |R_N(x)| \) for an \( x \in I \) is:

\[
|R_N(x)| \leq
\]
2. \[ D_x \cos(\ln x) = \]

3. \[ D_x 7^{(x^2)} = \]
4. \[
\int (\tan x) (\sec^7 x) \, dx =
\]

Remark: box your substitution box for more partial credit.
5. 

\[
\int x^2 \arctan x \, dx =
\]

Remark: box your substitution box for more partial credit.
6. \[ \int \frac{x^2}{\sqrt{4-x^2}} \, dx = \]

Remark: box your substitution box for more partial credit.
7. Let $R$ be the region enclosed by

$$y = x^2 \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0.$$  

Let $V$ be the volume of the solid obtained by revolving the region $R$ about the line $x = 3$.

7a. Make a rough sketch below of the region $R$, labeling the important points.

7b. Using the disk/washer method, express the volume $V$ as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

\[
V = \]

8. \[
\lim_{n \to \infty} \frac{3n^{5/2} + 7n^2 + 9}{-17n^{5/2} + 3n^2 - 9n - 18} =
\]

9. \[
\lim_{n \to \infty} \frac{8n^{15} - 7n^{10} + 19}{-5n^{13} + 6n^8 - 6n^5 + 9} =
\]
On problems 10 and 11b, check the correct box and then indicate your reasoning below. A correctly checked box without appropriate explanation will receive no points.

10. \[
\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}
\]

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent
11a. Let \( a_n = \frac{n^3(n!)}{(2n)!} \). Find \( \frac{a_{n+1}}{a_n} \). Simplify your answer so that no factorial sign (i.e., \(!\)) appears.

**answer:** \( \frac{a_{n+1}}{a_n} = \)

11b. \( \sum_{n=17}^{\infty} (-1)^n \frac{n^3(n!)}{(2n)!} \) 

- [ ] absolutely convergent  
- [ ] conditionally convergent  
- [ ] divergent
12. Consider the formal power series
\[ \sum_{n=1}^{\infty} \frac{(2x + 8)^n}{n} . \]
As we did in class, in the box below draw a diagram indicating for which \( x \)'s this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.
13. Let

\[ f(x) = (1 + x)^{3/2} \]

and \( a = 0 \).

Find the 3\(^{rd}\)-order Taylor polynomial of \( y = f(x) \) about (or at) \( a = 0 \).

\[ P_3(x) = \]
14. As in problem 13, let

\[ f(x) = (1 + x)^{3/2} \]

and \( a = 0 \).

Let \( f(x) = P_3(x) + R_3(x) \), where \( y = P_3(x) \) is the 3rd-order Taylor polynomial of \( y = f(x) \) about \( a = 0 \) and \( y = R_3(x) \) is the corresponding remainder (i.e., error) term.

Consider the interval \( I = (-0.5, 0.5) \) center about \( a = 0 \). Fix an \( x \in I \). Find a good upper bound for \(|R_3(x)|\).

\[ |R_3(x)| \leq \]

Remark: you only have to carry out the algebra as far as I indicated in class.