1. \[ D_x \left[ e^{3x^2 + 1} \right] = 6x e^{3x^2 + 1} \]

2. \[ D_x \left[ \ln (3x^2 + 17) \right] = \frac{6x}{3x^2 + 17} \text{ Variation of Exam 1 \#4} \]

3. \[ D_x \left( (1+x)^2x \right) = \left[ 2 \ln (1+x) + \frac{2x}{1+x} \right] (1+x)^{2x} \text{ Variation of Exam 1 \#5} \]

HINT: Use logarithmic differentiation.

4. The rate of decay of a radioactive substance is proportional to the amount of such substance present.

Today we have 50 grams of a radioactive substance. Given that one-third of the substance decays every 5 years, how much will be left \( t \) years from today? Clearly explain your notation.

\[ \text{Answer: } P(t) = 30 e^{-\frac{\ln(3)}{5} t} \text{ grams} \]

HINT: Your answer should have a \( t \) in it.

\[ \frac{dP}{dt} = -\frac{\ln(3)}{5} P(t) \Rightarrow P(t) = P_0 e^{-\frac{\ln(3)}{5} t} \]

\[ \frac{2}{5} (30) \Rightarrow P(5) \Rightarrow 30 e^{-\frac{\ln(3)}{5} (5)} \]

\[ e^\frac{5\ln(3)}{5} = e^{\ln(3)} \]

\[ 5\ln = \ln(3^5) \]

\[ \ln e^5 = \ln (3^5) \]

\[ 5 = \frac{1}{5} \ln (3^{\frac{5}{5}}) \]

5. \[ \int \sin 2x \, dx = -\frac{1}{2} (1 - \cos 2x) \text{ Example 2} \]

6. \[ \int \sin 2x \, dx = -\frac{1}{2} \cos 2x + x + C \text{ Example 2} \]

7. \[ \int \sin 2x \, dx = -\frac{1}{2} \cos 2x + x + C \text{ Example 2} \]

8. \[ \int \ln x \, dx = x \ln x - x + C \text{ Example 2} \]

9. \[ \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C \]

text book \[ \frac{1}{2} x^2 \cos x + 2 \sin x + 2 \cos x + C \text{ Example 4} \]
10. \[ \int \frac{x^2 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} \, dx = \frac{x^2}{2} + x + 2 \ln(x-1) - 2(x-1)^{-1} - 2 \ln(x+1) + C \]

HINT: \( x^3 - x^2 - x + 1 = (x-1)^3(x+1) \)

\[ \frac{x^4 - 2x^3 + 4x + 1}{x^3 - x^2 - x + 1} = \frac{4x}{x^3 - x^2 - x + 1} \]

\[ \frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \]

\[ \frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \]

\[ 4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \]

\[ (x-1) \Rightarrow 4 = 2B \Rightarrow B = 2 \]

\[ (x+1) \Rightarrow -4 = 4C \Rightarrow C = -1 \]

\[ 2(x-1)^2 \Rightarrow 4 = A \]

\[ A = 4 \]

\[ x = \frac{4}{x+1} + \frac{2}{(x-1)^2} + \frac{-1}{x+1} \]

\[ \frac{1}{x+1} \ \text{and} \ (x-1)^2 \]

11. \[ \lim_{x \to +\infty} \frac{1}{x^2} = 0 = 1 \]

Exam 2 Part B

12. \[ \int_0^\infty \frac{dx}{x+1} = \infty \text{ or } DNE \text{ or } diverges \]

Textbook 9.15 Part B

13. \[ \lim_{n \to +\infty} \frac{12n^{17} + 18n^7 - 10n}{4n^{15} + 10n} = \lim_{n \to +\infty} \frac{12n^{17}}{4n^{15}} \]

\[ = \lim_{n \to +\infty} \frac{12n^2}{4} \]

\[ = 0 + 0 - 0 \text{ or } \sum \text{ or diverges} \]

14. \[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \]

Absolutely convergent

15. \[ \sum_{n=1}^{\infty} \left( \frac{3}{2} \right)^n \frac{1}{n!} \]

Conditionally convergent

16. Consider the formal power series

As we did in class, in the box below draw a diagram indicating for which \( x \)'s this series is:

- Absolutely convergent
- Conditionally convergent
- Divergent

Of course, indicate your reasoning.

\[ \sum_{n=1}^{\infty} \frac{(x+5)^n}{n} \]

An Example from class

As we did in class, in the box below draw a diagram indicating for which \( x \)'s this series is:

- Absolutely convergent
- Conditionally convergent
- Divergent

Of course, indicate your reasoning.

\[ \sum_{n=1}^{\infty} \frac{(x+5)^n}{n} \]

As we did in class, in the box below draw a diagram indicating for which \( x \)'s this series is:

- Absolutely convergent
- Conditionally convergent
- Divergent

Of course, indicate your reasoning.
17. The third order Taylor polynomial of \( f(z) = (1+z)^{1/4} \) about \( a = 0 \) is:

\[
P_3(z) = 1 + \frac{3}{2} z + \frac{3}{8} z^2 - \frac{1}{16} \frac{3}{4} z^3
\]

but made easier:

\[
f(x) = (1+x)^{1/4}
\]

\[
f(0) = 1
\]

\[
f^{(1)}(x) = \frac{3}{2} (1+x)^{-3/4}
\]

\[
f^{(1)}(0) = \frac{3}{2}
\]

\[
f^{(2)}(x) = \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} (1+x)^{-7/4}
\]

\[
f^{(2)}(0) = \frac{3}{4}
\]

\[
f^{(3)}(x) = \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{3}{4} (1+x)^{-9/4}
\]

\[
f^{(3)}(0) = \frac{9}{4}
\]

18. Consider the function \( f(z) = \ln(2+z) \). Let \( a = 0 \). We know that we can write the function as

\[
f(z) = P_3(z) + R_3(z)
\]

where \( P_3 \) is the third order Taylor polynomial of \( f \) about \( a = 0 \) and \( R_3 \) is the corresponding remainder term.

18a: Find a formula for the remainder term \( R_3(z) \). Your answer should have a \( "c" \) in it, be sure to indicate where \( c \) lies.

\[
R_3(z) = \frac{-6(2+c)^{-4}}{4!} x^4
\]

where \( c \) is between 0 and 0.

\[
f(z) = \ln(2+z) \Rightarrow f^{(4)}(x) = -6 (2+z)^{-4}
\]

\[
\frac{1}{(0.5)^4} = \frac{1}{2^4}
\]

\[
| R_3(0.5) | = \frac{(0.5)^4}{4} \cdot \frac{1}{(2+c)^4} \leq \frac{(0.5)^4}{4} \cdot \frac{1}{(0.5)^4} = \frac{1}{4}
\]

18b: Find a good upper bound for \( |R_3(0.5)| \). Your answer should not have a \( "c" \) in it but you do not have to do arithmetic. Do you work on the back of the previous page.

\[
| R_3(0.5) | \leq \frac{1}{4}
\]
19. Using the method from class, sketch the graph of the polar equation \( r = 3 - 4 \sin \theta \).

\[
\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}
\]

\[
0 = 3 - 4\sin \theta \Rightarrow \sin \theta = \frac{3}{4}
\]

<table>
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<th>( \sin \theta )</th>
<th>( r = 3 - 4\sin \theta )</th>
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20. Consider the polar equation \( r = 4 \cos(3\theta) \).

20a. Using the method from class, sketch the graph of this polar equation.

Make your chart on the back of the previous page.

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<th>( r )</th>
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<td>-4</td>
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<tr>
<td>2\pi</td>
<td>4\pi</td>
<td>0</td>
</tr>
</tbody>
</table>

20b. Express the area enclosed by this polar equation as an integral (but you do not have to evaluate this integral).

\[
\text{Area} = 6 \cdot \frac{1}{2} \int_{\theta = 0}^{\frac{\pi}{6}} (4 \cos(3\theta))^2 \, d\theta \leq 48 \int_{\theta = 0}^{\frac{\pi}{6}} \cos^2(3\theta) \, d\theta
\]

\[
6 \cdot \frac{1}{2} \cdot 4^2 = 3 \cdot 16
\]