Recall Maple Lab Assignment H

For the sequences \( \{a_n\}_{n=1}^{\infty} \) given in problems 1 through 3, do the following.

(a) Using Maple, compute and plot the first 25 terms of the sequence.
(b) Using Maple, decide if the sequence converges or diverges.
   If the sequence converges\(^1\), find the limit of the sequence.
   If the sequence diverges to \( +\infty \), indicate so.\(^2\)
   If the sequence diverges to \( -\infty \), indicate so.\(^2\)

If you are having troubles with Maple, then see the examples on the activity sheet from class.

1. \( a_n = n \sin \left( \frac{1}{n} \right) \)
2. \( a_n = \left( 1 + \frac{1}{n} \right)^n \)
3. \( a_n = \ln (n) \)

Now, a followup to Maple Lab H.

Problems (i), (ii), and (iii) are to be done without the use of Maple (i.e., by hand).
You can think of these problems as verifying, by hand, what Maple told you in Lab H.

(i). Consider the function \( f: [1, \infty) \to \mathbb{R} \) given by \( f(x) = x \sin \left( \frac{1}{x} \right) \).
Find the limit of the function \( y = f(x) \) as \( x \) approached infinity.
This can also be phrased as find \( \lim_{x \to \infty} f(x) \) or as \( \lim_{x \to \infty} x \sin \left( \frac{1}{x} \right) \).
Hint: you might have to use L’Hopital’s rule.

(ii). Consider the function \( f: [1, \infty) \to \mathbb{R} \) given by \( f(x) = \left( 1 + \frac{1}{x} \right)^x \).
Find the limit of the function \( y = f(x) \) as \( x \) approached infinity.
This can also be phrased as find \( \lim_{x \to \infty} f(x) \) or as \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^x \).
Hint: you might have to use L’Hopital’s rule.

(iii). Explain how (in complete sentence(s)) you can use Theorem 4 on page 577 in the textbook along
with your solutions to parts (i) and (ii), to

a. find the limit of the sequence \( \left\{ n \sin \left( \frac{1}{n} \right) \right\}_{n=1}^{\infty} \) as \( n \) approaches \( \infty \), i.e., find \( \lim_{n \to \infty} n \sin \left( \frac{1}{n} \right) \)

and

b. find the limit of the sequence \( \left\{ \left( 1 + \frac{1}{n} \right)^n \right\}_{n=1}^{\infty} \) as \( n \) approaches \( \infty \), i.e., find \( \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n \).

\( \triangleright \) If needed, review L’Hopital’s rule. We already needed a working knowledge of L’H’s rule in \( \S 8.8 \).
The direct link to the review (of \( \S 4.5 \)) handout Indeterminate Forms and L’Hopital’s Rule is


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\(^1\)Recall this means that you can find a real number \( L \) so that \( \lim_{n \to \infty} a_n = L \). See textbook page 574. Here \( L \in \mathbb{R} \), that is, \( L \) is a finite real number. The number \( L \) is called the limit of the sequence \( \{a_n\}_{n=1}^{\infty} \).

\(^2\)For explanation of the terminology, see textbook page 575.