
Name: _______________________

- Read problems carefully. Show all work. No notes, calculator, or text.
- There are 15 points total.

1. §12.1, #9 b (4 points): Determine whether the points $D = (0, -5, 5)$, $E = (1, -2, 4)$, and $F = (3, 4, 2)$ lie on a straight line. Explain how you got your answer.

Solution: It suffices to calculate the distances between these points:

\[ d(D, E) = \sqrt{1 + 9 + 1} = \sqrt{11}, \quad d(E, F) = \sqrt{4 + 36 + 4} = 2\sqrt{11}, \quad d(F, D) = \sqrt{9 + 81 + 9} = 3\sqrt{11}. \]

Now, since $d(D, E) + d(E, F) = d(F, D)$, we see that the points lie on a line.

Alternative: Consider the vectors $\overrightarrow{DE} = \langle 1, 3, -1 \rangle$ and $\overrightarrow{EF} = \langle 2, 6, -2 \rangle$. Since $2\overrightarrow{DE} = \overrightarrow{EF}$, we have $\overrightarrow{DE} \parallel \overrightarrow{EF}$. These vectors share the point $E$, so the points $E, F, D$ are collinear.

2. §12.1, #13 (4 points): Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

Solution: The radius is the distance between the points, $\sqrt{1 + 25 + 4} = \sqrt{30}$. Hence, the equation is $(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30$.

3. §12.2, #25 (4 points): If $\vec{v}$ lies in the first quadrant and makes an angle of $\pi/3$ with the positive $x$-axis and $\|\vec{v}\| = 4$, find $\vec{v}$ in the form $a\vec{i} + b\vec{j}$.

Solution: The vector is

\[ \vec{v} = 4(\cos(\pi/3)\vec{i} + \sin(\pi/3)\vec{j}) = 4\left(\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}\right) = 2\vec{i} + 2\sqrt{3}\vec{j}. \]

4. §12.2, #35 (3 points): Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

Solution: The value $dy/dx$ at $x = 2$ gives the ratio of the $y$-component to the $x$-component of a vector parallel to the tangent line. Since $dy/dx = 2x$, this value is $4 = 4/1$, so we seek unit vectors parallel to $\vec{i} + 4\vec{j}$. We obtain $\pm \left(\frac{1}{\sqrt{17}}\vec{i} + \frac{4}{\sqrt{17}}\vec{j}\right)$. 