Math 241, Quiz 7. 2/27/12. Name: ____________________

• Read problems carefully. Show all work. No notes, calculator, or text.
• There are 15 points total.

1. §14.5, #11 (5 points): Use the chain rule to find \( \frac{\partial z}{\partial s} \) when

\[
z = e^r \cos \theta, \quad r = st, \quad \theta = \sqrt{s^2 + t^2}.
\]

Your final answer can be in terms of \( r, s, t, \) and \( \theta \).

Solution: We have

\[
\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r} \frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial s}.
\]

\[
\frac{\partial z}{\partial r} = e^r \cos \theta, \quad \frac{\partial r}{\partial s} = t, \quad \frac{\partial z}{\partial \theta} = -e^r \sin \theta, \quad \frac{\partial \theta}{\partial s} = (1/2)(s^2 + t^2)^{-1/2} \cdot 2s = \frac{s}{\sqrt{s^2 + t^2}}.
\]

It follows that

\[
\frac{\partial z}{\partial s} = e^r \cos \theta \cdot t - e^r \sin \theta \cdot \frac{s}{\sqrt{s^2 + t^2}}.
\]

2. §14.6, #13 (5 points): Find the directional derivative of \( g(p, q) = p^4 - p^2q^3 \) at the point \( P = (2, 1) \) in the direction of \( \mathbf{v} = \mathbf{i} + 3\mathbf{j} \).

Solution: We have

\[
\nabla g(p, q) = (4p^3 - 2pq^3)\mathbf{i} + (-3p^2q^2)\mathbf{j} \implies \nabla g(2, 1) = 28\mathbf{i} - 12\mathbf{j}.
\]

A unit vector in the direction of \( \mathbf{v} \) is

\[
\mathbf{u} = \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{1}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j}).
\]

It follows that

\[
D_{\mathbf{u}}g(2, 1) = (28\mathbf{i} - 12\mathbf{j}) \cdot \frac{1}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j}) = \frac{1}{\sqrt{10}}(28 - 36) = -\frac{8}{\sqrt{10}}.
\]

3. §14.6, #27 b (5 points): Find the direction in which the function \( f(x, y) = x^4y - x^2y^3 \) decreases fastest at point \( P = (2, -3) \).

Solution: We have

\[
\nabla f(x, y) = (4x^3y - 2xy^3, x^4 - 3x^2y^2) \implies \\
\nabla f(2, -3) = ((4)(8)(-3) - (2)(2)(-3)^3, 16 - (3)(4)(9)) = (-96 + 108, -92) = (12, -92).
\]

Hence the direction of fastest decrease is \( \langle -12, 92 \rangle = 4\langle -3, 23 \rangle \) (so one could also use \( \langle -3, 23 \rangle \).