Math 241, Quiz 10. 4/4/12.

Name: ___________________

• Read problems carefully. Show all work. No notes, calculator, or text.
• There are 15 points total.

1. §15.4, #25 (7 points): Use polar coordinates to set up a double integral expressing the volume of the solid which lies above the cone \( z = \sqrt{x^2 + y^2} \) and below the sphere \( x^2 + y^2 + z^2 = 1 \). Your answer should be in the form

\[
\int_{\alpha}^{\beta} \int_{r_1(\theta)}^{r_2(\theta)} f(r, \theta) \, dA
\]

with appropriate limits, \( f(r, \theta) \), and expression for \( dA \). I only require the setup. Do not evaluate the double integral.

Solution: The boundary of \( D \) is obtained as

\[
x^2 + y^2 + (\sqrt{x^2 + y^2})^2 = 1 \iff 2x^2 + 2y^2 = 1 \iff x^2 + y^2 = \frac{1}{2}.
\]

I.e., \( D \) is the region in the \( xy \)-plane enclosed by a circle of radius \( 1/\sqrt{2} \) and center \((0, 0)\). We have

\[
V = \iint_{x^2+y^2\leq 1/2} \left( \sqrt{1-x^2-y^2} - \sqrt{x^2+y^2} \right) \, dA = \int_{\theta=0}^{2\pi} \int_{r=0}^{1/\sqrt{2}} \left( \sqrt{1-r^2} - r \right) r \, dr \, d\theta.
\]

2. §15.4, #29 (8 points): Evaluate

\[
\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin(x^2 + y^2) \, dy \, dx
\]

by converting to polar coordinates.

Solution: We have

\[
\iint_{D} f(x, y) \, dA = \int_{0}^{\pi} \int_{0}^{3} \sin(r^2) r \, dr \, d\theta = \int_{0}^{\pi} \sin(0) \, d\theta + \int_{0}^{3} \sin(r^2) r \, dr = \frac{\pi}{2} \int_{u=0}^{9} \sin u \, du
\]

\[
= \frac{\pi}{2} \left( -\cos 9 + \cos 0 \right) = \frac{\pi}{2} \left( 1 - \cos 9 \right).
\]